GRADE - 9

LESSON – 9 Areas of Parallelograms and Triangles

Objective Type Questions

I. Multiple choice questions

1 Two figures are congruent, if they have the

a) same size b) same shape c) same area d) same shape and size

(d)

2. If A and B are two congruent figures, then

a) ar (A) > ar (B) b) ar (A) = ar (B) c) ar (A) < ar (B) d) none of these

b)

3. Given parallelogram ABCD and EBCF on the same base BC and between the same parallels BC and AF. Given ar (EBCF) = 15 square cm, then ar (ABCD) is

a) 30 sq.cm b) 7.5 sq.cm c) 15 sq.cm d) 5 sq.cm

Sol : Parallelograms on the same base and between the same parallels are equal in area.

$\Rightarrow$ ar (ABCD) = ar (EBCF)

Since ar (EBCF) =15 sq.cm, so

ar (ABCD) = 15 sq.cm

$\therefore$ Correct option is (c)

4. If ar (Parallelogram ABCD) = 25 cm^2 and on same base CD a ΔBCD is given such that ar (ΔBCD) = xcm^2 then value of x is

a) 25 cm^2 b) 12 cm^2 c) 12.5 cm^2 d) 50 cm^2

Sol : If a triangle and a parallelogram are on the same base and between the same parallels, then the area of the triangle is equal to half the area of the parallelogram.

$\Rightarrow$ ar (ΔBCD) $= \frac{1}{2}\, ar\,(ABCD)$

$\Rightarrow$ $x = \frac{1}{2} \times 25 \Rightarrow x = 12.5\ cm^2$ $\therefore$ Correct option (c)

5. Two parallelograms are on equal bases and between the same parallels. The ratio of their areas is [NCERT Exemplar]

   a) 1 : 2 b) 1 : 1 c) 2 : 1 d) 3 : 1

   Sol : (b)

6. ABCD is a quadrilateral whose diagonal AC divides it into two parts, equal in area, then ABCD [NCERT Exemplar]

   a) is a rectangle

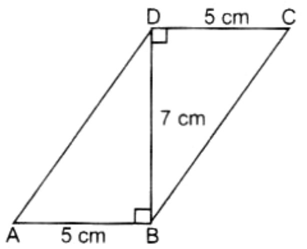
   b) is always a rhombus

   c) is a parallelogram

   d) need not be any of (a), (b), or (c)

   (d)

7. In the given figure, ABCD is parallelogram. Calculate the area of parallelogram ABCD.



   Sol :Area of parallelogram ABCD

   = base x altitude = AB x DB

   = 5 x 7 = 35 cm^2

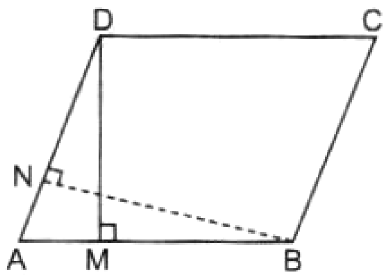
8. Find the area of a rhombus, the length of whose diagonals are 16cm and 12cm respectively.

   Sol : Area of rhombus $= \frac{1}{2} \times d_1 \times d_2$

   $= \frac{1}{2} \times 16 \times 12\ cm^2$

   $= 96\ cm^2$

9. In parallelogram ABCD, AB = 8 cm and the altitudes corresponding to sides AB and AD are DM = 6cm and BN = 10cm respectively. Find AD.



Sol : Area of parallelogram = base x altitude

$\therefore$ AD X BN = AB X DM

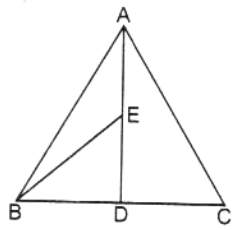
AD x 10 = 8 x 6

AD = $\frac{8 \times 6}{10} = \frac{48}{10} = 4.8cm$

II. Multiple choice questions

1. Given a triangle ABC and E is mid-point of median AD of ΔABC. If ar (ΔBED) = 20 cm^2. Then ar (ΔABC) is.

   a) 10 cm^2 b) 5 cm^2 c) 60 cm^2 d) 80 cm^2



Sol : Since BE is the median of ΔABD,so

ar (ΔBED) = $\frac{1}{2}$ ar (ΔABD)

$\Rightarrow$ 20 = $\frac{1}{2}$ ar (ΔABD)

$\Rightarrow$ ar(ΔABD) = 40 cm^2

Since AD is the median of ΔABC, so

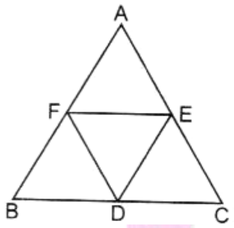
ar(ΔABC) = 2ar(ΔABD)

$\Rightarrow$ ar(ΔABC) = 2 $\times$ 40 = 80 cm^2

$\therefore$ *Correct option is* (d)

2. The mid-point of the sides of a triangle along with any of the vertices as fourth point make a parallelogram of area equal to half area of triangle

a) True b) False



Sol: We have

ar(Δ DEF) $= \frac{1}{4}$ ar(ΔABC)

Now, ar(BDEF) = ar(ΔDEF) + ar(ΔFBD)

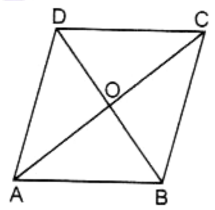
= 2 ar(ΔDEF)

= 2 x $\frac{1}{4}$ ar(ΔABC)

= $\frac{1}{2}$ ar(ΔABC)

∴ correct option is (a).

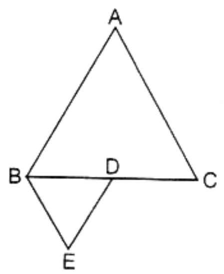
3. In the given figure, ABCD is a parallelogram in which diagonals AC and BD intersect at O. If ar(||gm ABCD) is 68 cm^2, then find ar(ΔOAB).



Sol: We have ar(ΔOAB) = $\frac{1}{4}$ × ar(||gm ABCD)

= $\frac{1}{4}$ x 68cm^2 = 17 cm^2

4. ABC and BDE are two equilateral triangles such that D is the mid-point of BC. Prove that ar(ΔBDE). = $\frac{1}{4}$ ar(ΔABC)



Sol: Let the side of triangle, BC = a

$\Rightarrow$ BD $= \frac{a}{2}$

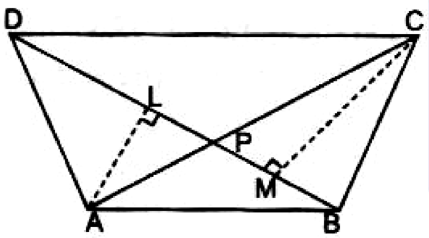
$\therefore$ ar(Δ BDE) $= \frac{\sqrt{3}}{4}\left(\frac{a}{2}\right)^2 = \frac{\sqrt{3}}{4} \times \frac{a^2}{4}$

$= \frac{1}{4}(\frac{\sqrt{3}}{4} \times a^2)$

$\therefore$ ar(Δ BDE) $= \frac{1}{4}$ar(Δ ABC). Hence proved.

I. Short answer questions

1. Diagonals AC and BD of a quadrilateral ABCD intersect each other at P. Show that ar(ΔAPB) x ar(ΔCPD) = ar(ΔAPD) x ar(ΔBPC)



Sol. We have

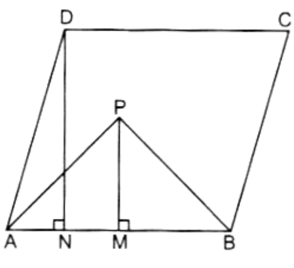
ar (ΔAPD) x ar (ΔBPC)

$= \left(\frac{1}{2} \times AL \times DP\right) \times \left(\frac{1}{2} \times CM \times BP\right)$

$= \left(\frac{1}{2} \times BP \times AL\right) \times \left(\frac{1}{2} \times DP \times CM\right)$

= ar (ΔAPB) x ar (ΔCPD) Hence Proved.

2. If P is any point in the interior of a parallelogram ABCD, then prove that area of (ΔAPB) is less than half the area of the parallelogram



Sol. Given: P is any point in the interior of parallelogram ABCD

To prove : ar (ΔAPB) < $\frac{1}{2}$ (ar ‖gm ABCD)

Construction: Draw DN ⊥ AB and PM ⊥ AB.

Proof : ar (‖ gm ABCD) = AB x DN

ar (ΔAPB) = $\frac{1}{2}$ (AB X PM)

Now, PM < DN

⇒ AB x PM < AB x DN

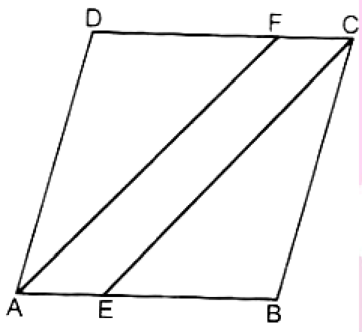
⇒ $\frac{1}{2}$ (AB X PM) < $\frac{1}{2}$ (AB X DN)

⇒ ar (ΔAPB) < $\frac{1}{2}$ ar (‖ gm ABCD)

Hence proved

II. Short answer questions

1. ABCD is a parallelogram. E is a point on BA such that BE = 2EA and F is a point on DC such that DF = 2FC. Prove that AECF is a parallelogram whose area is one-third of the area of parallelogram ABCD.



Sol. Given : A parallelogram ABCD. E is a point on BA ∋BE= 2EA and F is a point on DC such that DF= 2FC.

To prove: (i) AECF is a parallelogram.

(ii) ar (‖ gm AECF) = $\frac{1}{3}$ar (‖ gm ABCD)

Proof : BE = 2EA and DF = 2FC

⇒ AB – AE = 2AE and DC – FC = 2FC

⇒ AB = 3AE and DC = 3FC

$\Rightarrow$ AE = $\frac{1}{3}$AB and FC = $\frac{1}{3}$DC

$\Rightarrow$ AE = FC ($\because$ AB = CD)

$\therefore$ AE $\parallel$ FC such that AE = FC

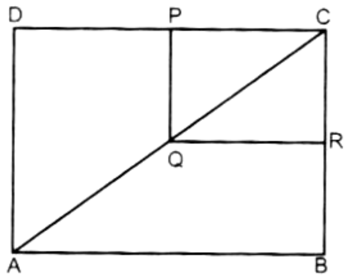
$\therefore$ AEFC is a parallelogram.

Parallelograms ABCD and AECF have the same altitude and AE = $\frac{1}{3}$AB

$\therefore$ $ar(\parallel \text{gm AECF}) = \frac{1}{3} ar(\parallel \text{gm ABCD})$

Hence proved.

3. ABCD and PQRC rectangles. Q is mid-point of AC. Show that P is the mid-point of DC and R is the mid-point of BC. Also, fine the ratio of ar(ABCD) and ar (PQRC)



Sol. Given : ABCD and PQRC are rectangles. Q is mid-point of AC

To prove : P,R are mid-points of DC, BC respectively and find ar(ABCD) : ar (PQRC)

Proof : In Δ CAB, Q is the mid-point of AC.

QR $\parallel$ AB

($\because$ ABCD and PQRC both are rectangles)

$\Rightarrow$ R is the mid-point of BC. (By converse of mid-point theorem)

Again in Δ CAB, Q and R are the mid-points of AC and BC respectively.

$\Rightarrow$ QR = $\frac{1}{2}$ AB (By mid-point theorem)

$\Rightarrow$ QR = $\frac{1}{2}$ DC

($\because$AB = DC, Opposite sides of a rectangle)

In Δ CAD, Q is the mid –point of AC.

Again, PQ $\parallel$ DA

$\Rightarrow$ P is the mid-point of DC

(By converse of mid-point theorem)

Again in Δ CAD, Q and P are the mid-points of AC and DC respectively

⇒ PQ = $\frac{1}{2}$ DA (By mid-point theorem)

⇒ PQ = $\frac{1}{2}$ CB ----(ii)

(∵ DA = CB, opposite sides of a rectangle)

Now, ar (ABCD) = DC X CB -------(iii)

ar (PQRC) = QR X PQ = $\left(\frac{1}{2}\,DC\right)$ X $\left(\frac{1}{2}\,CB\right)$

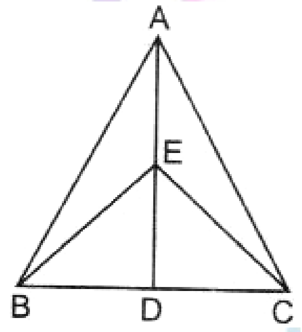
= $\frac{1}{4}$ DC x CB = $\frac{1}{4}$ (ar ABCD)

⇒ $\frac{\text{ar (PQRC)}}{\text{ar(ABCD)}} = \frac{1}{4}$ i.e. = 1:4

Hence, ar (PQRC) : ar (ABCD) = 1:4

III. Short answer questions

1. D and E are the mid-points of BC and AD respectively of ΔABC. If area of ΔABC = 20cm^2 find area of ΔEBD.



Sol. ∵ D is the mid-point of BC

∴ AD is the median of ΔABC

⇒ ar (ΔABD) = $\frac{1}{2}$ ar (ΔABC)

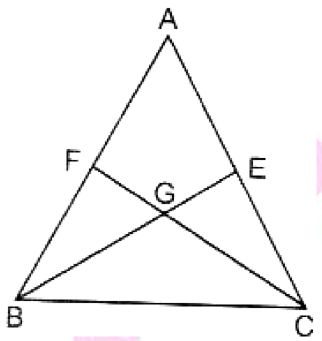
(∵ Median of a triangle divides it into two triangles of equal areas)

⇒ ar (ΔABD) = $\frac{1}{2}$ x 20cm^2 = 10cm^2

Also, BE is the median of ΔABD.

ar (ΔEBD) = $\frac{1}{2}$ ar (ΔABD) = $\frac{1}{2}$ x 10 = 5 cm^2

2. The medians BE and CF of a ΔABC intersect at G. Prove that ar (ΔGBC) = ar (AFGE)



Sol. Given : Medians BE and CF of ΔABC intersect at G.

To prove : ar (ΔGBC) = ar (AFGE)

Proof : We have ar (ΔFBC) = $\frac{1}{2}$ ar (ΔABC) -------(i)

(∵ Median CF divides ΔABC into two triangles of equal areas)

Similarly, ar(ΔEBC) = $\frac{1}{2}$ ar (ΔABC) ------(ii)

From (i) and (ii) we get

ar (ΔFBC) = ar (ΔEBC) ----(iii)

Subtracting ar (ΔBGC) from both sides of (iii) , we get

ar (ΔFBC) - ar (ΔBGC) = ar (ΔEBC) - ar (ΔBGC)

⇒ ar (ΔFGB)= ar (ΔEGC) ---(iv)

Also, ar (ΔABE) = ar (ΔBEC)

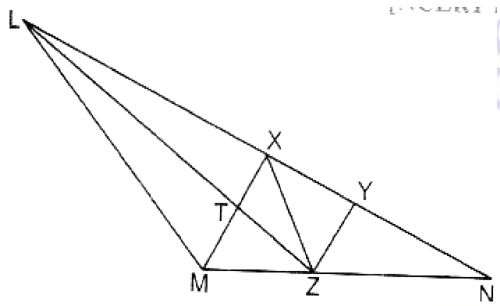
(∵ BE is median of ΔABC)

⇒ ar(ΔBFG) + ar (AFGE) = ar (ΔBGC) + ar (ΔGEC)

⇒ ar(AFGE) = ar (ΔBGC) Using (iv)

Hence proved

3. In the given figure, X and Y are point on side LN of the Δ LMN such that LX = XY =YN. Through X, a line is drawn parallel to LM to meet MN at Z. Prove that ar(ΔLZY) = ar(MZYX)



Sol. Given : LX = XY =YN. And XZ || LM

To prove : ar (ΔLZY) = ar (MZYX)

Proof : XZ || LM

ΔLXZ and ΔXMZ are on the same base XZ and between the same parallels XZ and LM.

∴ ar (ΔLXZ) = ar (ΔXMZ)

Adding ar (Δ XYZ) onboth sides we get

ar (Δ LXZ) +ar (Δ XMZ)

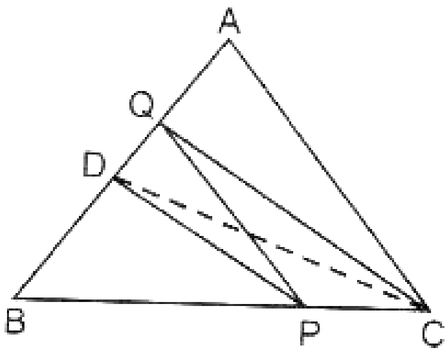
Adding ar (Δ XYZ) on both sides, we get

ar (ΔLXZ) + ar (ΔXYZ) = ar (ΔXMZ) + ar (ΔXYZ)

∴ ar (ΔLZY) = ar (ΔMZYX) Hence proved.

IV. Short answer questions

4. In ΔABC, D is the mid-point of AB and P is any point on BC. If CQ || PD meets AB in Q in the given figure, then prove that ar(ΔBPQ) = $\frac{1}{2}$ ar(ΔABC) [NCERT Exemplar]



Sol: Given A $\Delta ABC, D$ is the mid-point of AB and P is any point on BC and CQ||PD meets AB in Q

To prove: ar (ΔBPQ) = $\frac{1}{2}$ ar (ΔABC)

Construction: Join D and C

Proof : since CD is the median of ΔABC.

∴ ar (ΔBCD) = $\frac{1}{2}$ ar (ΔABC)

(∵ Median of a triangle divides it into triangles of equal areas) ....(i)

DP|| CQ [Given]

∴ ar (ΔDPQ) = $\frac{1}{2}$ ar (ΔDPC)

[Triangles are on the same base DP and between the same parallels DP and CQ]

Adding ar (ΔDBP) on both sides, we get

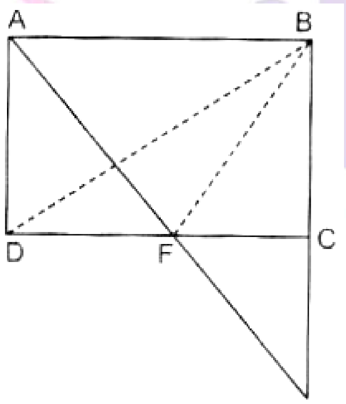
ar (ΔDPQ) + ar (ΔDBP) = ar (ΔDPC) + ar (ΔDBP)

⇒ ar (ΔBPQ) = ar (ΔBCD) ....(ii)

From (i) and (ii) , we get

ar (ΔPQB) = $\frac{1}{2}$ ar (ΔABC) Hence proved.

5. In the given figure. ABCD is a parallelogram in which BC is produced to E such that CE = BC, AE intersects CD at F, If area of ΔBDF = $3cm^2$ Find the area of parallelogram ABCD



Sol. In ΔADF and ECF,

∠ADF = ∠ECF [Alternate interior angles]

AD = CE (∵ AD =BC and BC = CE)

∠DFA = ∠CFE (Vertically opposite angles)

ΔADF ≅ Δ ECF (AAS congruence rule)

⇒ ar(ΔADF) = ar (Δ ECF)

Also DF = CF (CPCT)

∴ F is the mid-point of DC

⇒ BF is the median in ΔBCD

⇒ ar (ΔBCD) = 2ar (ΔBDF)

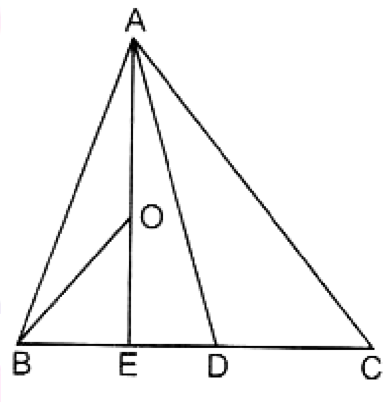
(∵ Median of a triangle divides it into two triangles of equal areas)

⇒ ar (ΔBCD) = 2 x 3 cm^2 = 6cm^2

∴ ar (|| gm ABCD) = 2ar (ΔBCD)

= (2 x 6) cm^2 = 12 cm^2

6. D is the mid-point of side BC of ΔABC and E is the mid-point of BD. If O is the mid-point of AE then prove that ar (Δ BOE) = $\frac{1}{8}$ ar ((ΔABC)



Sol. Given : D, E and O are mid-points of BC, BD and AE respectively

To prove : ar (Δ BOE) = $\frac{1}{8}$ ar ((ΔABC)

Proof : since AD and AE are the medians of ΔABC and ΔABD respectively.

∴ ar (ΔABD) = $\frac{1}{2}$ ar (ΔABC) ---(1)

and ar (ΔABE) = $\frac{1}{2}$ ar (ΔABD) ---(2)

Also, OB is the median of ΔABE

∴ ar (ΔBOE) = $\frac{1}{2}$ ar (ΔABE) ___(3)

From (i), (ii) and (iii) we get ar (ΔBOE) = $\frac{1}{2}$ ar (ΔABE)

= $\frac{1}{2}$ × $\frac{1}{2}$ ar (ΔABD)

= $\frac{1}{4}$ ar (ΔABD)

= $\frac{1}{4}$ × $\frac{1}{2}$ ar (ΔABC)

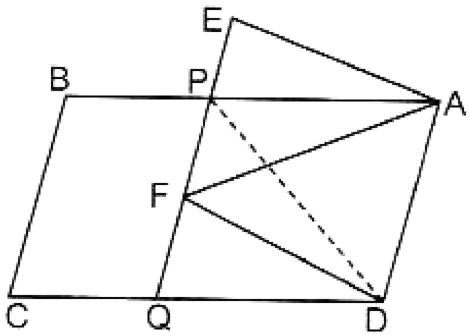
= $\frac{1}{8}$ ar (ΔABC) Hence proved

7. In the given figure, ABCD and AEFD are two parallelogram. Prove that

i) PE = FQ

ii) ar(ΔPEA) = ar (ΔQFD)

iii) ar(ΔAPE) : ar (ΔPFA) = ar (ΔQFD) : ar (ΔPFD)



Sol. Given : ABCD And AEFD are two parallelograms

To prove :

i) PE = FQ

ii) ar(ΔPEA) = ar (ΔQFD)

iii) ar(ΔAPE) : ar (ΔPFA) = ar (ΔQFD) : ar (ΔPFD)

Proof : i) In ΔAPE and ΔDQF

∠APE = ∠DQF (Corresponding angles)

AE = DF (Opposite sides of a parallelogram)

∠AEP = ∠DFQ (Corresponding angles)

Δ APE ≅ Δ DQF (AAS congruence rule)

⇒ PE = QF (CPCT)

(ii) ar (ΔPEA) = ar (ΔQFD) ----(i)

(∴ Congruent triangles are equal in areas)

(iii) Δ PFA = Δ PFD are on the same base PF and between the same parallels PQ and AD

∴ ar (Δ PFA) = ar (Δ PFD) ---(2)

Dividing (i) by (ii), we get

$$\frac{ar\ (\Delta\ \mathrm{APE})}{ar\ (\Delta\ \mathrm{PFA})} = \frac{ar\ (\Delta\ \mathrm{QFD})}{ar\ (\Delta\ \mathrm{PFD})}$$

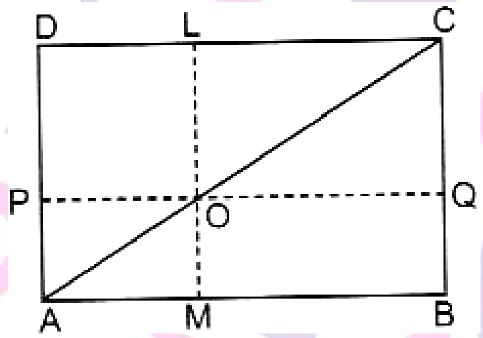
⇒ ar (ΔAPE) : ar (ΔPFA)

=ar (ΔQFD) : ar (ΔPFD)

Hence proved

I. Long answer questions

1. In the given figure, ABCD is a parallelogram. O is any point on AC. PQ ∥ AB and LM ∥ AD. Prove that ar(∥ gm DLOP) = ar (∥ gm BMOQ)



Sol. Given : ABCD is a parallelogram and point O lies on AC. PQ ∥ ABandLM ∥ AD

To prove : ar (parallelogram DLOP)

= ar (parallelogram BMOQ)

Proof :∵ Diagonal of a ar parallelogram divides it into two triangles of equal areas

∴ ar (ΔADC) = ar (ΔABC)

⇒ ar (ΔAPO) + ar(∥ gm DLOP) + ar (ΔLOC)

= ar (ΔAOM) + ar (∥ gm BMOQ) + ar (ΔOQC) ----(i)

∵ AO and OC are diagonals of parallelogram

AMOP and OQCL respectively

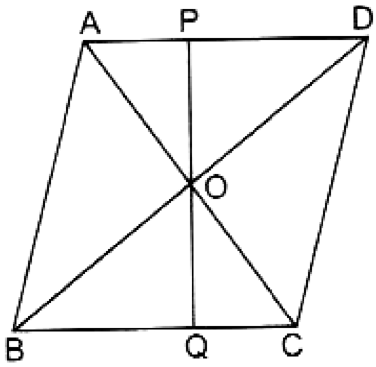
∴ ar (ΔAPO) = ar (ΔAMO) ----(ii)

ar (ΔOLC) = ar (ΔOQC) ----(iii)

Subtracting (ii) and (iii) from (i) we get

ar(∥ gm DLOP) = ar (∥ gm BMOQ) Hence proved

2. The diagonals of a parallelogram ABCD intersect at a point O. Through O, a line is drawn to intersect AD at P and BC at Q. Show that PQ divides the parallelogram into two parts of equal area. [NCERT Exemplar]



Sol. Given : A parallelogram ABCD in which diagonals AC and BD intersect at O. Through O, a line is drawn to intersect AD at P and BC at Q.

To prove : ar (APQB) = ar (PQCD)

$= \frac{1}{2}$(ǁ gm ABCD)

Proof : Diagonals of a parallelogram divides it into two triangles of equal area.

ar (ΔABC) = ar (ΔACD)

ar (ABQO) + ar (COQ) = ar (CDPO) + ar (ΔAOP)----(i)

In ΔAOP and ΔCOQ, we have

∠AOP = ∠ COQ (Vertically opposite angles)

OA = OC (Diagonals of a parallelogram bisect each other)

∠OAP = ∠ OCQ (Alternate interior angles)

∴ ΔAOP ≅ ΔCOQ (ASA congruence rule)

As congruent triangles are equal in areas,

⇒ ar (ΔAOP) = ar (ΔCOQ)

From (i) and (ii) we get

ar (ABQO) + ar (ΔAOP) = ar (CDPO) + ar (ΔCOQ)

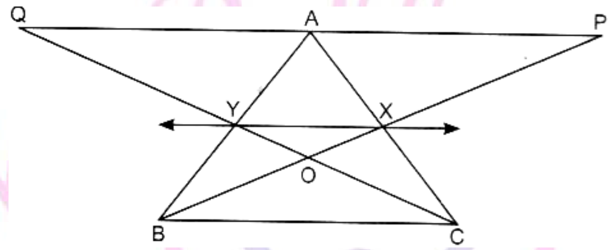
ar (ABQO) =ar (CDPQ)

ar (APQB) =ar (PQCD)

Hence proved.

II. Long answer questions

1. **In the given figure, X and Y are the mid-points of AC and AB respectively, QP ∥ BC and CYQ and BXP are straight lines. Prove that ar(ΔABP) = ar(ΔACQ)**



Sol. Given: X and Y are mid-points of AC and AB respectively. QP ∥ BC and CYQ and BXP are straight lines.

To prove: ar (ΔABP) = ar (ΔACQ)

Prove: X and Y are the mid-points of AC and AB respectively.

∴ By mid-point theorem

XY ∥ BC

Triangles BYC and BXC are on the same base BC and between the same parallel XY and BC.

∴ ar(ΔBYC) = ar(ΔBXC)

Subtracting ar(ΔBOC) from both sides, we get

ar(ΔBYC) − ar(ΔBOC) = ar(ΔBXC) − ar(ΔBOC)

⟹ ar(ΔBOY) = ar(ΔCOX)

Adding ar(ΔXOY) on both sides, we get

ar(ΔBOY) + ar(ΔXOY) = ar(ΔCOX) + ar(ΔXOY)

⟹ ar(ΔBXY) = ar(ΔCXY) .... (i)

∵ Parallelogram XYAP and Parallelogram XYAQ are on the same base XY and between the same parallels XY and between the same parallels XY and PQ

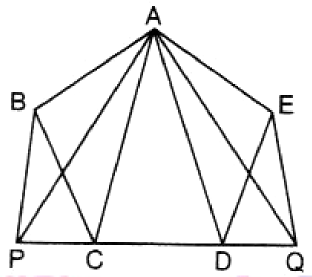
∴ ar(ΔXYAP) = ar(ΔXYQA) .... (ii)

Adding (i) and (ii), we get

ar(ΔBXY) + ar(XYAP) = ar(ΔCXY) + ar(XYQA)

⟹ ar(ΔABP) = ar(ΔACQ) Hence proved.

2. **In the given figure, ABCDE is any pentagon. BP drawn parallel to AC meets DC produced at P and EQ drawn parallel to AD meets CD produced at Q. Prove that ar(ABCDE) = $\mathrm{ar}(\Delta APQ)$ [NCERT Examplar]**



Sol. Given: ABCDE is any pentagon, BP ∥ AC and EQ ∥ AD

To Prove : ar (ABCDE) = ar (ΔAPQ)

Proof : ΔABC and ΔAPC are on the same base AC and between the same parallels BP and AC

∴ ar (ΔABC) = ar (ΔAPC) ……….. (i)

Similarly, ΔAED and ΔAQD are on the same base AD and between the same parallels AD and EQ

∴ ar (ΔAED) = ar (ΔAQD) …….(ii)

Adding (i) and (ii),we get

ar (ΔABC) = ar (ΔAED) = ar (ΔAPC) = ar (ΔAQD) …. (iii)

Adding ar(ΔACD) on both sides of (iii), we get

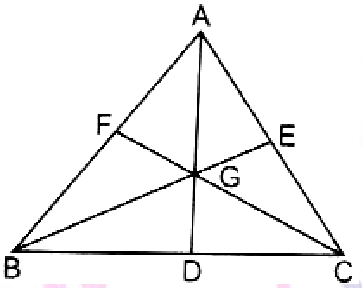
ar (ΔABC) + ar (ΔAED) + ar (ΔACD)

= ar (ΔAPC) + ar (ΔAQD) + ar (ΔACD)

⟹ ar (ABCDE) = ar (ΔAPQ)

Hence proved.

3. If the medians of a (ΔABC) intersect at G, show that $ar(\Delta AGC) = ar(\Delta AGB) = ar(\Delta BGC) = \frac{1}{3}(\Delta ABC)$.



Sol. Given : A ΔABC in hich medians AD, BE and CF intersect each other at G.

To prove: ar (ΔAGC) = ar (ΔAGB) = ar (ΔBGC)

$= \frac{1}{3}$ar (ΔABC)

Proof : In ΔABC, AD is the median.

As a median of a triangle divides it into two triangles of equal areas,

$\therefore$ ar(ΔABD) = ar(ΔACD) ……(i)

In ΔGBC, GD is the median,

$\therefore$ ar(ΔGBD) = ar(ΔGCD) …..(ii)

Subtracting (ii) from (i), we get

$\therefore$ ar(ΔABD) − ar(ΔGBD) = ar(ΔACD) − ar(ΔGCD)…….(iii)

ar(ΔAGB) = ar(ΔAGC)

Similarly,

ar(ΔAGB) = ar(ΔBGC) …….. (iv)

From (iii) and (iv) , we get

ar(ΔAGB) = ar(ΔBGC)

= ar(ΔAGC) …….. (v)

But ar(ΔAGB) + ar(ΔBGC) + ar(ΔAGC)

= ar(ΔABC) …………(vi)

From (v) and (vi) , we get

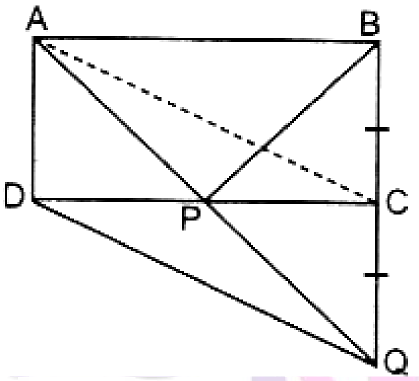
3ar(ΔAGB) = ar(ΔABC)

$ar(\Delta AGB) = \frac{1}{3} ar(\Delta ABC)$

Hence, $ar(\Delta AGB) = ar(\Delta AGC) = ar(\Delta BGC)$

$= \frac{1}{3} ar(\Delta ABC)$ Hence proved.

4. In the given figure, ABCD is a parallelogram. Prove that $ar(\Delta BCP) = ar(\Delta DPQ)$, if BC = CQ **[CBSE 2016]**



Sol. Given :A parallelogram ABCD in which BC = CQ

To prove: $ar(\Delta BCP) = ar(\Delta DPQ)$,

Construction : Join AC

Proof : since ΔAPC and ΔBPC are on the same base PC and between the same parallels PC and AB.

∴ $ar(\Delta APC) = ar(\Delta BPC)$ …… (i)

Since ABCD is a parallelogram.

AD =BC

⟹ AD = CQ (∵ BC = CQ)

Now, AD || CQ and AD= CQ

Thus in quadrilateral ADQC, one pair of opposite sides is equal and parallel

∴ ADCQ is a parallelogram.

⟹ AP = PQ and CP = DP

[Diagonals of a parallelogram bisect each other]

Now, in ΔAPC and DPQ

AP = PQ [Proved above]

$\angle APC = \angle DPQ$ [Vertically opposite angles]

$PC = PD$ [Proved above]

$\therefore \Delta APC \cong \Delta DPQ$ [SAS congruence rule]

$\Rightarrow ar(\Delta APC) = ar(\Delta DPQ)$ ....(ii)

From (i) and (ii), we get

$ar(\Delta BCP) = ar(\Delta DPQ)$

Hence proved.