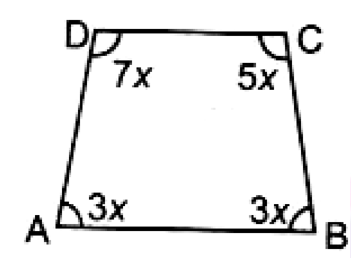
Grade IX

Lesson : 8 [Quadrilaterals]

Objective Type Questions

I. Multiple choice questions

1. The value of x in the given figure is



a) 10^0 b) 20^0 c) 30^0 d) 40^0

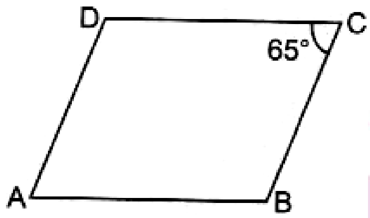
Sol : b

2. Which of the following is not a parallelogram?

a) trapezium b) square c) rectangle d) rhombus

Sol : a

3. In the given figure, ABCD is a parallelogram. If $\angle C = 65^0$, then $(\angle B + \angle D)$ is equal to



a) 180^0 b) 115^0 c) 155^0 d) 230^0

Sol : Since ABCD is a parallelogram, so opposite angles are equal,

Thus $\angle B = \angle D \; and \; \angle A = \angle C = 65^0$

using angle sum property of a quadrilateral, we have

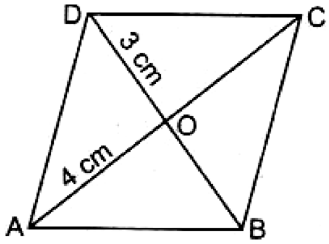
$\angle A + \angle B + \angle C + \angle D = 360^0$

$\Longrightarrow 65^0 + \angle B + 65^0 + \angle D = 360^0$

$\Longrightarrow \angle B + \angle D = 360^0 - 130^0 = 230^0$

$\therefore Correct\ option\ is\ (d)$

4. In the given figure, ABCD is a rhombus, AO = 4cm and DO = 3cm. Then the perimeter of the rhombus is



a) 18cm b) 20cm c) 21cm d) 22 cm

Sol : Since ABCD is a rhombus, so diagonals AC and BD bisect each other right angles at O.

so∠AOD = 90^0

Now, $AD^2 = AO^2 + OD^2$ [$\because$ *Pythagora theorem*]

$\Rightarrow AD^2 = (4)^2 + (3)^2$

$\Rightarrow AD^2 = 16 + 9$

$\Rightarrow AD^2 = 25$

$\Rightarrow AD = 5\,cm$

Since all sides of a rhombus are equal, so AB = BC = CD = DA = 5cm

So, perimeter of rhombus = AB + BC + CD + DA

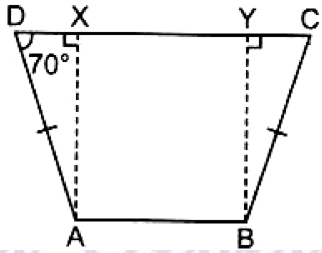
= 5 + 5 + 5 + 5 = 20cm

$\therefore$ *Correct option is* (*b*)

5. Given a trapezium ABCD, in which $AB||CD$ and AD = BC. If $\angle D = 70^0$, then $\angle C$ will be

a) 70^0 b) 110^0 c) 20^0 d) *none of these*

Sol : Draw AX ⊥ DC and BY ⊥ DC



In △AXD and △BYC, we have

AD = BC [Given]

$\angle AXD = \angle BYC$ [Each 90^0]

$AX = BY$ [Distance between parallel sides]

So, $\triangle AXD \cong \triangle BYC$, [RHS congruence rule]

Thus $\angle D = \angle C$ [CPCT]

Hence, $\angle C = 70^0$

$\therefore$ *Correct option is* (a)

6.Three angles of quadrilateral are $75^0, 90^0, 75^0$, Find the fourth angle [NCERT Exemplar]

Sol : As we know that sum of four angles of quadrilateral is 360^0

Let fourth angle be x.

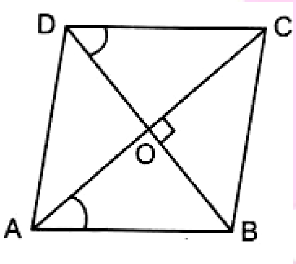
$\therefore 75^0 + 90^0 + 75^0 + x = 360^0$

$\Rightarrow x = 360^0 - 240^0 = 120^0$

$\therefore$ *Fourth angle* $= 120^0$

7. Diagonals AC and BD parallelogram ABCD intersect at O. If $\angle BOC = 90^0$ and $\angle BDC = 50^0$ find $\angle OAB$

Sol : In a parallelogram ABCD, O is point of intersection of diagonals AC and BD.



$\angle BDC = \angle DBA$ [Alternate angles as $AC||CD$]

$\angle BDC = 50^0$ [Given]

$\Rightarrow \angle DAB = 50^0$

We have $\angle BOC = \angle OBA + \angle OAB$

[Exterior angle is equal to sum of two interior opposite angles]

$\Rightarrow 90° = 50° + \angle OAB$

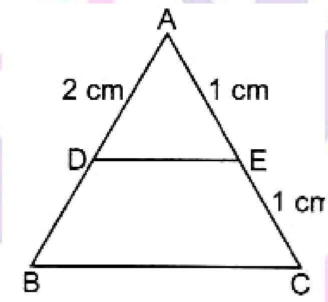
$\Rightarrow \angle OAB = 90^0 - 50^0 = 40^0$

8. Can all the angles of a quadrilateral be acute angles? Give reason for your answer. [NCERT Exemplar]

Sol : No, all the angles of quadrilateral cannot be acute angles. If all the angles of quadrilateral will be acute. The sum of all the four angles will be less than 360^0 which is not possible

II. Multiple choice questions

1. In the given figure, find BD, if $DE||BC$

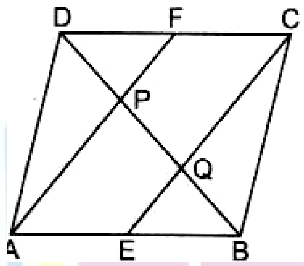


a) 2cm b) 1cm c) 3cm d) none of these

Sol : a) by the converse of the Mid-point theorem.

2. In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively. AF and CE meet the diagonal BD of length 12cm at P and Q, then length of PQ is
   a) 6cm b) 4 cm c) 3 cm d) 5 cm

Sol. In a parallelogram ABCD, we know that if E and F are the mid-points of sides AB and CD respectively, then the line segments AF and EC trisect the diagonal BD.



So DP = PQ = OB

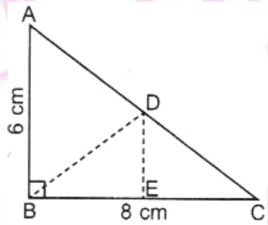
Now PQ $= \frac{1}{3}$BD $= \frac{1}{3}$x 12 = 4cm

$\therefore$ *Correct option is* (b)

3. A In $\triangle ABC$, right angled at B, Side AB =6cm and side BC = 8cm. D is mid-point of AC. Then length of BD is

a) 10cm b) 4 cm c) 3 cm d) 5 cm

Sol : By converse of mid-point theorem, we get E is mid-point of BC.



Also $\angle DEC = \angle ABC = 90^0$ [Corresponding Angle[

Now, $\triangle CED \cong \triangle BDE$ [SAS Congruence rule]

So, CD = BD [CPCT]

$\Rightarrow$ CD = BD = $\frac{1}{2}AC$ [$\because$ D is mid-point of AC]

$\Rightarrow$ BD = $\frac{1}{2} \times 10 = 5cm$

[$\because AC = \sqrt{(AB)^2 + (BC)^2}$, Pythagoras theorem]

$\therefore Correct\ option\ is\ (d)$

4. D and E are the mid-points of the sides AB and AC of ΔABC and O is any point on side BC. O is joined to A. If P and Q are the mid-points of OB and OC respectively, then DEQP is [NCRT Examplar]

a) a square b) a rectangle c) a rhombus d) a parallelogram

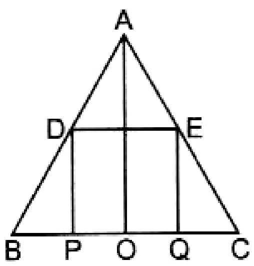
Sol : Using mid-point theorem, we have

$DP||EQ$

And DP = EQ = $\frac{1}{2}AO$

So, DEQP is a parallelogram,

$\therefore Correct\ option\ is\ (d)$



I. Short answer questions

1. **If one angle of a parallelogram is 36^0 less than twice its adjacent angle, then find the angles of parallelogram** **[CBSE 2016]**

Sol : Let one angle of parallelogram be x.

Its adjacent angle is $(180^0 - x)$

As per question,

$x = 2\ (180^0 - x) - 36^0$

$\Rightarrow \quad x = 360^0 - 2x = 36^0$

$\Rightarrow \quad 3x = 324^0$

$\Rightarrow \quad x = \frac{324^0}{3} = 108^0$

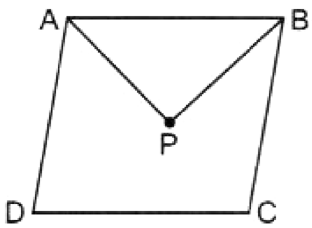
$\Rightarrow \quad Adjacent\ angle = 180^0 - 108^0 = 72^0$

Hence, the angles of parallelogram are 108^0, 72^0, 108^0, 72^0.

2. **In a parallelogram, show that the angle bisectors of two adjacent angles intersect at right angles.**

Sol: **Given:** ABCD is a parallelogram such that angle bisectors of adjacent angles A and B intersect at point P.

To prove: $\angle APB = 90^0$



Proof : we have

$\angle A + \angle B = 180^0$

[$AD \,||\, BC$ and $\angle A$ and $\angle B$ are consecutive interior angles]

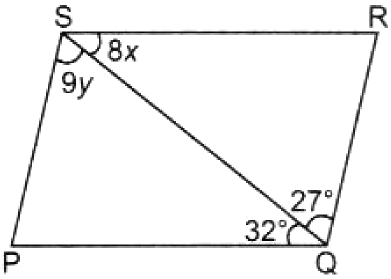
$\frac{1}{2}\angle A + \frac{1}{2}\angle B = 90^0$

But $\frac{1}{2}\angle A + \frac{1}{2}\angle B + \angle APB = 180^0$ [Sum of angles of a triangle is 180^0]

$\Rightarrow \quad 90^0 + \angle APB = 180^0$

$\Rightarrow \quad \angle APB = 90^0$ Hence proved

3. In the given figure, PQRS is a parallelogram. Find the values of x and y.



Sol : Here, PQRS is a parallelogram

As $PQ||RS$

∴ $8x = 32^0$ [Alternate interior angles]

As $PS||QR$

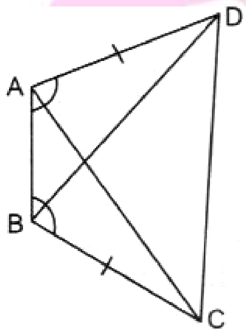
And $9y = 27^0$ [Alternate interior angles]

⟹ $x = \frac{32^0}{8}$ and $y = \frac{27^0}{9}$

⟹ $x = 4^0$ and $y = 3^0$

II. Short answer questions

1. **In the given figure, ABCD is a quadrilateral in which AD = BC and ∠DAB = ∠CBA**



Prove that : (i) △ABD ≅ △ BAC

(ii) BD = AC

Sol : Given : ABCD is a quadrilateral in which AD = BC and ∠DAB = ∠CBA

To prove:

(i) $\triangle ABD \cong \triangle BAC$

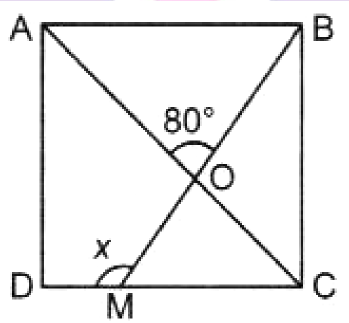
(ii) $BD = AC$

Proof: (i) In - $\triangle ABD$ and $\triangle BAC$

$AD = BC$	[Given]
$\angle DAB = \angle CBA$	[Given]
$AB = AB$	[Common]
$\therefore \triangle ABD \cong \triangle BAC$	[SAS congruence rule]
(ii) $BD = AC$	[CPCT]

Hence proved

2. **In the given figure, ABCD is a square, A line BM intersects CD at M and diagonal AC at O such that $\angle AOB = 80^0$. Find the value of x.**



Sol. As diagonal of a square bisects the opposite angles,

$\angle BAO = \frac{1}{2}\angle BAD = \frac{1}{2} \times 90^0 = 45^0$

$\angle BAC = \angle ACD$ [Alternate interior angles]

$\therefore \angle ACD = \angle BAC = 45^0$ .....(i)

Also $\angle AOB = \angle MOC = 80^0$ .....(ii) [Vertically opposite angles]

Now, $x = \angle MOC + \angle OCM$

[Exterior angle is equal to sum of two interior opposite angles]

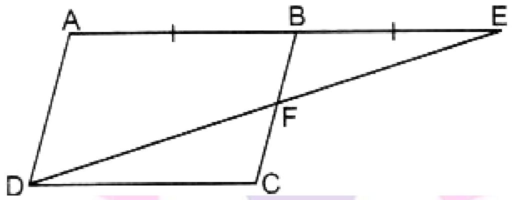
$\therefore x = 80^0 + 45^0 = 125^0$

III. Short answer questions

1. ABCD is a parallelogram. AB is produced to E so that BE = AB. Prove the ED bisects BC.

Sol : Given : ABCD is a parallelogram. AB is produced to E such that BE = AB

To prove : ED bisects BC.



i.e. BF = FC

Construction: Join D to E which intersects BC at F.

Proof : We have

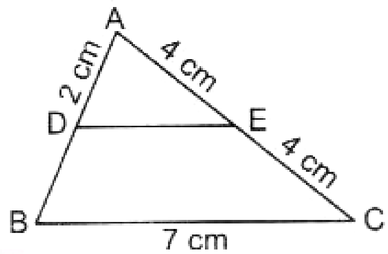
	AB = DC	[Opposite sides of parallelogram]
But	AB = BE	[Given]
∴	BE = DC	

In ΔBEF and ΔCDF,

	BE = DC	[Proved above]
	∠BEF = ∠CDF	[Alternative interior angles]
	∠BEF = ∠CFD	[Vertically opposite angles]
∴	ΔBEF ≅ ΔCDF,	[AAS congruence rule]
∴	BF = FC	[CPCT]

∴ $ED\ bisects\ BC$. Hence proved.

2. In the given figure, $DE||BC$ Find BD.



Sol : As AE = EC = 4cm

∴ E is mid-point of AC

Also DE || BC (given)

∴ By the converse of mid–point theorem, we have D is mid-point of AB

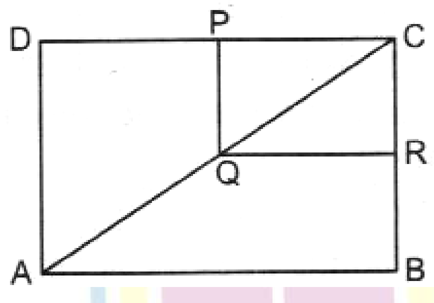
⇒ AD = BD

⇒ BD = 2cm [As given AD = 2cm]

IV. Short answer questions

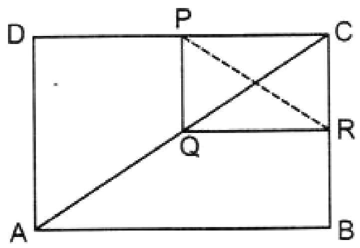
1. In the given figure, ABCD and PQRC are rectangles and Q is the mid–point of AC. Prove that :

i) DP = PC **ii) PR $= \frac{1}{2}$ AC**



Given : ABCD and PQRC are two rectangle and Q is the mid-point of AC

To prove: i) DP = BC ii) PR $= \frac{1}{2}$ AC



Proof : (i) In Δ ACD,

Q is the mid-point of AC

$\angle ADC = \angle QPC = 90^0$

(Each angle of rectangle is right angle)

But these are corresponding angles.

$\Rightarrow PQ \parallel DA$

$\therefore$ P is the mid–point of CD

i.e. DP = PC

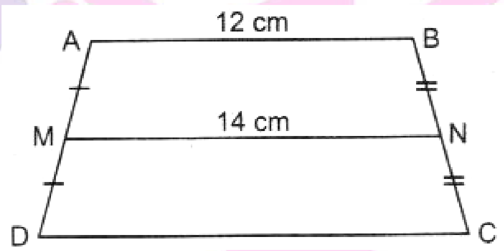
ii) We have QC = PR (Diagonals of rectangle are equal)

and $QC = = \frac{1}{2} AC$ (Given)

$\therefore PR = \frac{1}{2} AC$ Hence proved

2. ABCD is a trapezium in which AB∥DC. M and N are the mid–points of AD and BC respectively. If AB =12cm and MN =14cm, find CD. [HOTS]

Sol: Here, ABCD is a trapezium in which, AB∥DC and M and N are the mid – point of AD and BC respectively.



Since the line segment joining the mid-points of non-parallel sides of trapezium is half of the sum of the lengths of its parallel sides,

$\Rightarrow MN = \frac{1}{2}(AB + CD)$

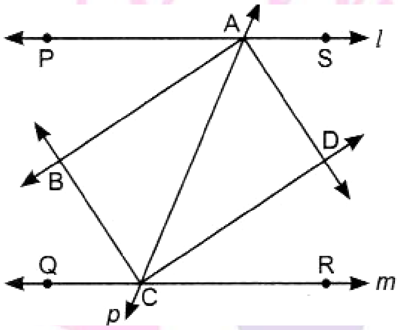
$\Rightarrow 14 = \frac{1}{2}(12 + CD)$

$\Rightarrow 28 = 12 + CD$

$\Rightarrow CD = 28 - 12 = 16$ cm

I. Long answer questions

1. **Two parallel lines l and m are intersected by a transversal p. Show that the quadrilateral formed by the bisectors of interior angles is a rectangle.**



Sol. Given BA, BC, DC, DA are bisectors of ∠PAC, ∠QCA, ∠ACR and ∠SAC respectively.

To prove : ABCD is a rectangle.

Proof : We have

∠PAC = ∠ACR

[Alternate interior angles as $l || m$ and p is transversal]

$\frac{1}{2}$∠PAC = $\frac{1}{2}$∠ACR

⟹ ∠BAC = ∠ACD

[As BA and DC are bisectors of ∠PAC and ∠ACR respectively]

But these are alternate angles. This shows that $AB || CD$

Similarly, $BC || AD$

⟹ *Quadrilateral ABCD is a parallelogram* .....(i)

Now, ∠PAC + ∠CAS = 180^0 [Linear pari axiom]

⟹ $\frac{1}{2}$∠PAC + $\frac{1}{2}$∠CAS = 90^0

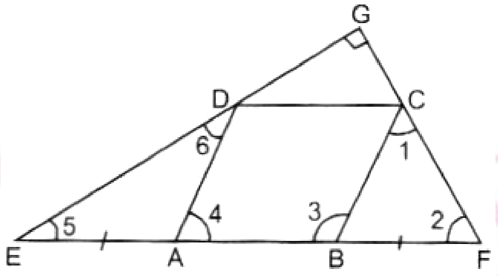
⟹ ∠BAC + ∠CAD = 90^0

⟹ ∠BAD = 90^0 ...... (ii)

From (i) and (ii) we can say that ABCD is a rectangle

Hence proved.

2. **ABCD is a rhombus and AB is produced to E and F such that AE = AB = BF. Prove that ED and FC are perpendicular to each other.**



Sol. Given : ABCD is a rhombus, AB produced to E and F such that AE = AB = BF

Construction: Join ED and CF and produce it to meet at G.

To prove $ED \perp FC$

Proof : AB is produced to points E and F such that

AE = AB = BF ….. (i)

Also, since ABCD is a rhombus

AB = CD = BC = AD ….. (ii)

Now, in $\triangle BCF$,

BC = BF [From (i) and (ii)]

$\Rightarrow \angle 1 = \angle 2$

$\angle 3 = \angle 1 + \angle 2$ [Exterior angle]

$\angle 3 = 2\angle 2$ …..(iii)

Similarly, AE= AD

$\angle 5 = \angle 6$

$\Rightarrow \angle 4 = \angle 5 + \angle 6 = 2\angle 5$ …..(iv)

Adding (iii) and (iv), we get

$\angle 4 + \angle 3 = 2\angle 5 + 2\angle 2$

$\Rightarrow 180^0 = 2(\angle 5 + \angle 2)$

[$\because \angle 4$ *and* $\angle 3$ *are consecutive interior angles*]

$\Rightarrow \angle 5 + \angle 2 = 90^0$

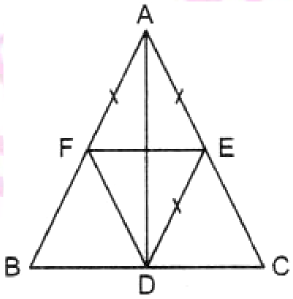
$\therefore EG \perp FC$, Now in ΔEGF

$\angle 5 + \angle 2 + \angle EGF = 180^0$

$\Rightarrow \angle EGF = 90^0$ Hence proved.

I. Long answer questions

1. In ΔABC is isosceles with AB = AC, D, E and F are the mid-point of sides BC, CA and AB respectively. Show that the line segment AD is perpendicular to the line segment EF and is bisected by it.



Sol : Given ΔABC is isosceles with AB = AC, D, E and F are the mid-point of BC, CA and AB respectively.

To Prove : $AD \perp EF$ and is bisected by it.

Construction: Join D, E and F and AD

Proof : we have

$DE \parallel AB$ and $DE = \frac{1}{2}AB$ .....(i)

And $DF \parallel AC$ and $DF = \frac{1}{2}AC$ .....(ii)

(Line segment joining mid-points of two sides of a triangle is parallel to the third side and is half of it.)

AB = AC ....(iii)

$\therefore$ $AF = \frac{1}{2}AB$, $AE = \frac{1}{2}AC$ .....(iv)

From (i), (ii), (iii), and (iv), we get

DE = DF = AF = AE

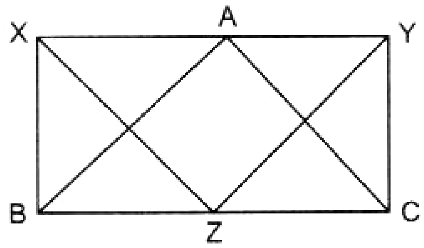
And also, $DF \parallel AE$ and $DE \parallel AF$

$\Rightarrow$ DEAF is a rhombus.

Since diagonals of a rhombus bisect each other at right angles,

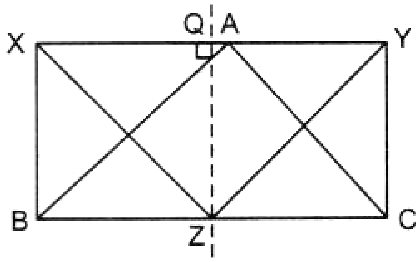
$\therefore$ $AD \perp EF$ and is bisected by it.

2. **In the given figure, BX and CY are perpendicular to a line through the vertex A of ΔABC and Z is the mid-point of BC. Prove that XZ = YZ [HOTS, CBSE 2015]**



Sol : Given BX and CY are perpendiculars to a line XAY

To prove: XZ = YZ



Construction : Draw ZQ ⊥ XY

Proof : We have

BX ⊥ XY [Given]

And CY ⊥ XY [Given]

Also, ZQ ⊥ XY [By construction]

BX ∥ ZQ ∥ CY

(Perpendiculars on same line are parallel to each other)

Now, As BX ∥ ZQ ∥ CY and Z is mid-point of BC

By mid-point theorem, we have Q is mid-point of XY.

In Δ**X**QZ and ΔYQZ,

XQ = QY [Q is mid-point of XY]

$\angle XQZ = \angle YQZ = 90^0$ [By construction]

QZ = QZ [Common]

∴ ΔXQZ ≅ ΔYQZ [SAS congruence rule]

⇒ XZ = YZ [CPCT]

Hence proved.