Grade IX

Lesson : 7 Triangles

CONGRUENCE OF TRIANGLES

CONGRUENT FIGURES

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Only when they have exactly

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Same size and same shape

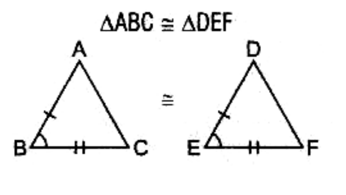
- Two equal angles
- Two line segments of equal length
- Circles of same radii
- Two triangles if they have equal corresponding sides and equal corresponding angles
- Squares of same sides

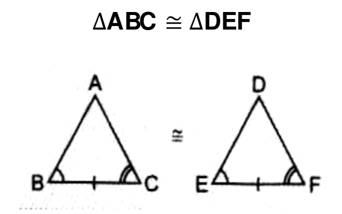
Example: In ΔABC and ΔDEF If $AB = DE$, $BC = EF$, $CA = FD$ and $\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$, then $\Delta ABC \cong \Delta DEF$

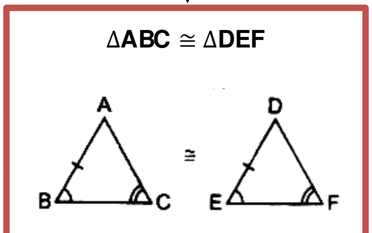
↓

Criteria for congruence of triangles

SAS (Side –Angle- Side) Congruence rule	ASA (Angle - Side - Angle) Congruence rule	AAS (Angle –Angle- Side) Congruence rule
Two Sides and the included angle	Two angles and included side	Two angles and one pair of corresponding sides
In ΔABC and ΔDEF ➢ $AB = DE$, $\angle B = \angle E$, and $BC = EF$	In ΔABC and ΔDEF ➢ $\angle B = \angle E$, and $BC = EF$ ➢ $\angle C = \angle F$	In ΔABC and ΔDEF ➢ $\angle B = \angle E$, $\angle C = \angle F$ $AB = DE$ or $AC = DF$







SOME MORE CRITERIA FOR CONGRUENCE OF TRIANGLES

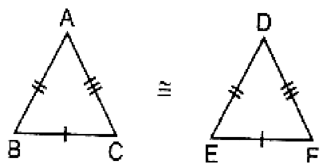
TWO MORE CRITERIA FOR CONGRUENCE OF TRIANGLES

SSS
(Side – Side - Side)
Congruence rule

Equal corresponding sides of two triangles

In ΔABC and ΔDEF
- AB = DE
- AC = DF
- BC = EF

$\Delta ABC \cong \Delta DEF$



$\Delta ABC \cong \Delta DEF$

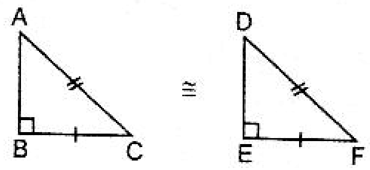
SSS Congruence rule: If three sides of one triangle are equal to three sides of another triangle, then the two triangles are congruent.

RHS
(Right Angle - Hypotenuse
Side - Congruence Rule)

In two right – angled triangles corresponding side and hypotenuse

In ΔABC and ΔDEF with $\angle B = \angle E = 90^0$,
- BC = EF
- AC = DF

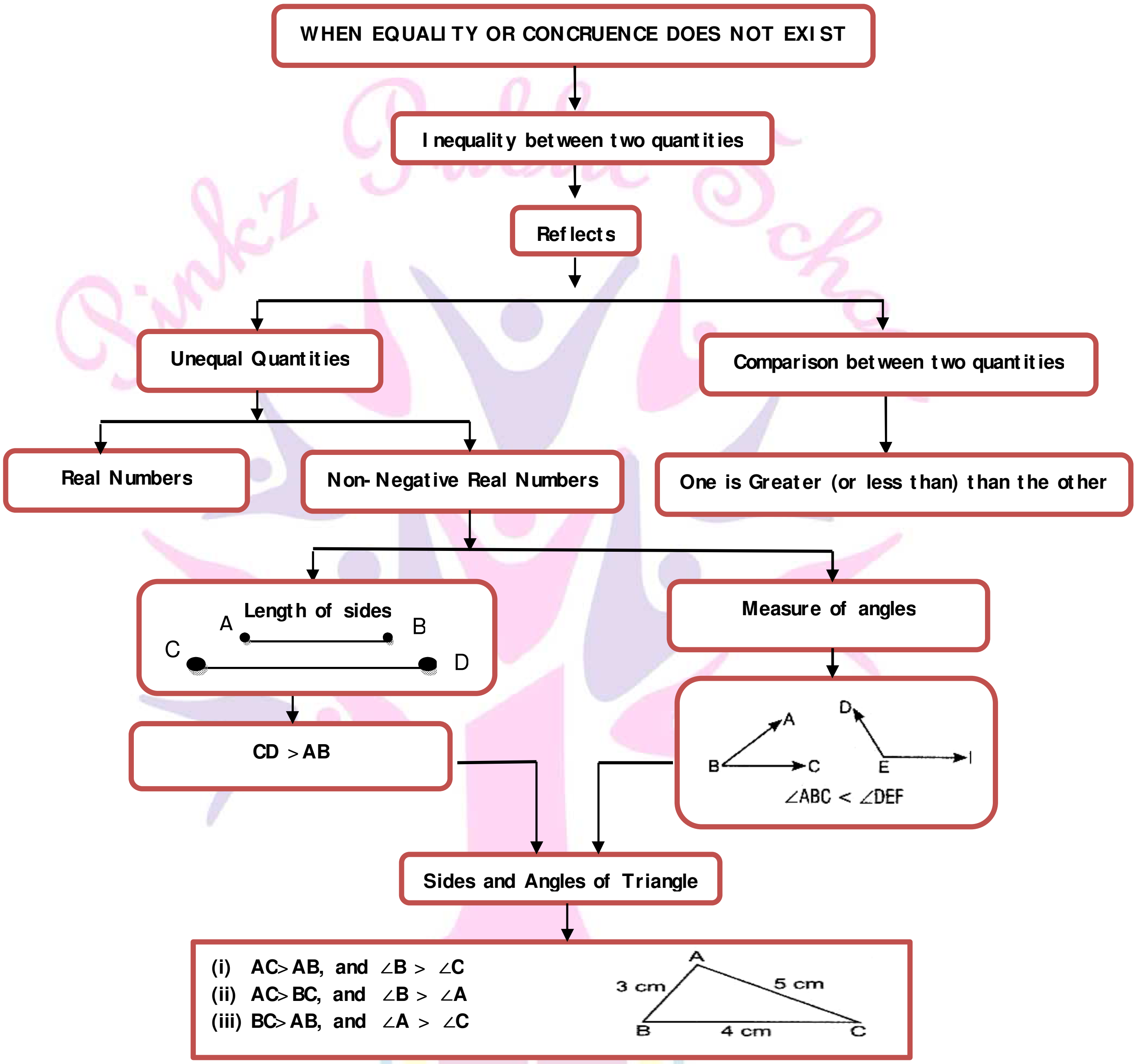
$\Delta ABC \cong \Delta DEF$



$\Delta ABC \cong \Delta DEF$

RHS Congruence rule: If in two right triangles the hypotenuse and one side of one triangle are equal to the hypotenuse and one side of the other triangle, then the two triangles are congruent.

INEQUALITIES IN A TRIANGLE



(i) If two sides of a triangle are unequal, the angle opposite to the longer side is larger [or greater]

(ii) In any triangle, the side opposite to the larger (greater) angle is longer.

(iii) The sum of any two sides of a triangle is greater than the third side

(a) $AB + BC > CA$ (b) $BC + CA > AB$ (c) $CA + AB > BC$

This gives us

(a) $AB > CA - BC$, i.e., $CA - BC < AB$ (b) $BC > AB - CA$, i.e., $AB - CA < BC$

(c) $CA > BC - AB$, i.e., $BC - AB < CA$

(iv) Of all line segments that can be drawn to a given line from a point not lying on it. The perpendicular line segment is the shortest.

Objective Type Questions

I. Multiple choice questions

1. **In Two triangles, ABC and PQR, $\angle A = 30^0$, $\angle B = 70^0$, $\angle P = 70^0$, $\angle Q = 80^0$ and AB = RP, then**

   a) $\Delta ABC \cong \Delta PQR$ b) $\Delta ABC \cong \Delta QRP$ c) $\Delta ABC \cong \Delta RPQ$ d) $\Delta ABC \cong \Delta RQP$

   Sol. (c)

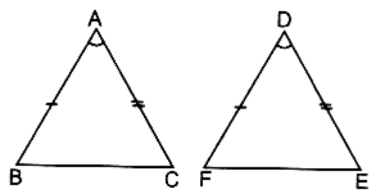
2. **If two sides and an angle of one triangle are equal to two sides and an angle of another triangle, then two triangles must be congruent**

   a) True b) False

   Sol: (b) Angles must be included angels

3. **In ΔABC and ΔDEF AB = FD and $\angle A = \angle D$. Write the third condition for which two triangles are congruent by SAS congruence rule.**

   Sol: By SAS congruence rule, the arms of equal angle must also be equal.



   Hence, AB = FD

   $\angle A = \angle D$

   So AC = DE

   $\Rightarrow$ $\Delta ABC \cong \Delta DFE$ [SAS congruence rule]

4. **It is given that $\Delta ABC \cong \Delta FDE$ and AB = 6 cm, $\angle B = 80^0$ and $\angle A = 40^0$. what is length of side DF of ΔFDE and its $\angle E$?**

   Sol. Given $\Delta ABC \cong \Delta FDE$

   Now, corresponding parts of congruent triangles are equal

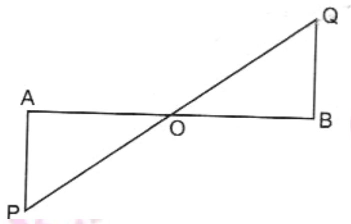
   So, DF = AB = 6 cm

   $\angle E = \angle C$

   $= 180^0 - (80^0 + 40^0) = 60^0$

5. In the given figure, O is the mid-point of AB and $\angle BQO = \angle APO$, Show that $\angle OAP = \angle OBQ$.

[CBSE 2014]



Sol. Given (i) O is mid-point of AB

(ii) $\angle BQO = \angle APO$

To prove $\angle OAP = \angle OBQ$

Proof : In ΔOAP and ΔOBQ,

$OA = OB$ [O is mid-point of AB]

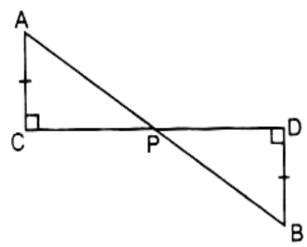
$\angle APO = \angle BQO$ [Given]

$\angle AOP = \angle BOQ$ [Vertically opposite angles]

$\Rightarrow$ $\Delta OAP \cong \Delta OBQ$ [ASA congruence rule]

$\Rightarrow$ $\angle OAP = \angle OBQ$ [CPCT] Hence proved.

6. In the given figure, CA and DB are perpendiculars to CD and CA = DB, show that PA = PB.



Sol. Given (i) $CA \perp CD$ (ii) $DB \perp CD$ (iii) $CA = DB$

To prove : PA = PB

Proof : In ΔCPA and ΔDPB

$\angle ACP = \angle BDP$ [Each 90^0]

$\angle CPA = \angle DPB$ [Vertically opposite angles]

$CA = DB$ [Given]

$\Rightarrow$ $\Delta CPA \cong \Delta DPB$ [AAS congruence rule]

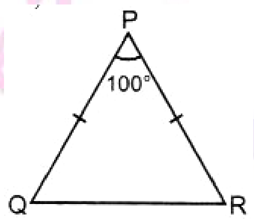
$\Rightarrow$ $PA = PB$ [CPCT] Hence proved.

II. Multiple choice questions

1. In a triangle PQR, if $\angle QPR = 100^0$ and $PQ = PR$, then $\angle R$ and $\angle Q$ respectively are

a) $80^0, 70^0$ b) $80^0, 80^0$ c) $70^0, 80^0$ d) $40^0, 40^0$

Sol : Since in an isosceles triangle, angles opposite to equal sides are equal, so



$\angle PRQ = \angle PQR$ $(\because Given\ PQ = PR)$

Now, in ΔPQR,

$\angle QPR + \angle PQR + \angle PRQ = 180^0$
$(\because Angle\ sum\ property\ of\ triangle)$

$\Rightarrow \quad 100^0 + \angle PQR + \angle PQR = 180^0$

$\Rightarrow \quad 2\angle PQR = 80^0 \Rightarrow \angle PQR = \frac{80°}{2} = 40^0$

So, $\angle PRQ = 40^0$

Hence $\angle R$ and $\angle Q$ resepectively are 40^0, 40^0

$\therefore Correct\ option\ is\ (d)$

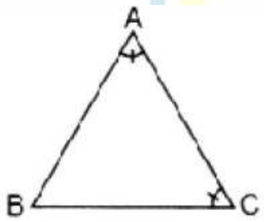
2. In ΔPQR $\angle R = \angle P$ and $QR = 4cm\ and\ PR = 5cm$. Then the length of PQ is [NCRT Exemplar]

a) 4 cm b) 5 cm c)2 cm d) 2.5 cm

Sol : (a)

3. In ΔABC, $\angle A = \angle C$ and BC = 4 cm and AC =3cm, what is length of side AB?

Sol : The sides opposite to equal angles are equal.

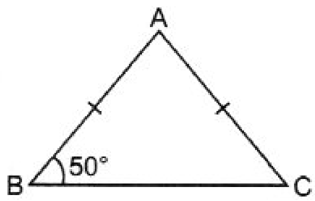


$\therefore AB = BC$ [Given $\angle A = \angle C$]

$\Rightarrow \quad AB = 4\ cm$

4. In the given figure of ΔABC, $AB = AC$, What will be $\angle BCA$?

Sol : Since in an isosceles triangle, angles opposite to equal sides are equal



Hence, $\angle BCA = \angle ABC$ [Given AB = AC]

$\Rightarrow$ $\angle BCA = 50^0$

5. Two angles measures $a - 60^0$ and $123^0 - 2a$. If each one is opposite to equal sides of an isosceles triangle, then find the value of a.

Sol. Since angles opposite to equal sides of an isosceles triangle are equal

Therefore $a - 60^0 = 123^0 - 2a$.

$\Rightarrow$ $3a = 123^0 + 60^0 = 183^0$

$\Rightarrow$ $a = \frac{183^0}{3} = 61^0$

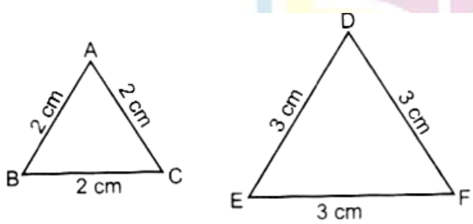
III. Multiple choice questions

1. Choose the correct statement from the following

(a) a triangle has two right angles
(b) all the angles of a triangle are more than 60^0
(c) an exterior angle of a triangle is always greater than the opposite interior angles
(d) all the angles of a triangle are less than 60^0

Sol. (c)

2. For the given triangles, write the correspondence, if congruent.



a) $\Delta ABC \cong \Delta DEF$ b) $\Delta ABC \cong \Delta EFD$ c) $\Delta ABC \cong \Delta FDE$ d) not congruent

Sol : (d)

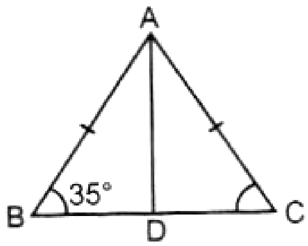
3. In ΔPQR, if $\angle R > \angle Q$, then [NCERT Exemplar]

a) $QR > PR$ b) $PQ > PR$ c) $PQ < PR$ d) $QR < PR$

Sol : (b)

4. In the given figure, AD is the median, then $\angle BAD$ is

a) 35^0, b) 70^0 c) 110^0 d) 55^0



Sol : In BAD and CAD, BD = DC

[$\therefore$ AD is median, so D is mid-point of BC]

AB = AC [Given]

AD = AD [Common]

$\Rightarrow \quad \Delta BAD \cong \Delta CAD$ [SSS $congruence$ $rule$]

$\Rightarrow \quad \angle BAD = \angle CAD$ [$CPCT$]

Also, $\angle ABC = \angle ACB = 35^0$

($\because AB = AC$ and $\angle B = 35^0$)

Now, in BAC, we have

$\angle BAC + \angle ABC + \angle ACB = 180^0$

($\because$ $Angle$ sum $property$ of a $triangle$)

$\Rightarrow \quad \angle BAC + 35^0 + 35^0 = 180^0$

$\Rightarrow \quad \angle BAC = 110^0$

$\Rightarrow \quad 2\angle BAD = 110^0$

$\Rightarrow \quad \angle BAD = 55^0$

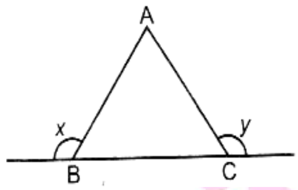
$\therefore$ Correct option is (d)

5. $\angle x$ and $\angle y$ are exterior angles of a ΔABC, at the points B and C respectively. Also $\angle B > \angle C$, then relation between . $\angle x$ and $\angle y$ is

a) $\angle x > \angle y$ b) $\angle x = \angle y$ c) $\angle x < \angle y$ d) none of these

Sol : we have $\angle x = \angle A + \angle C$ ($\because$ *Exterior angle property*)

and $\angle y = \angle A + \angle B$ ($\because$ *Exterior angle property*)



also, $\angle B > \angle C$ [Given]

$\Rightarrow \angle A + \angle B > \angle A + \angle C$

$\Rightarrow \angle y > \angle x$

$\Rightarrow \angle x < \angle y$

$\therefore$ Correct option is (c)

6. Two sides of a triangle are of lengths 5cm and 1.5cm. The length of the third side of the triangle cannot be [NCERT Exemplar]

a) 3.6cm b) 4.1 cm c) 3.8 cm d) 3.4 cm

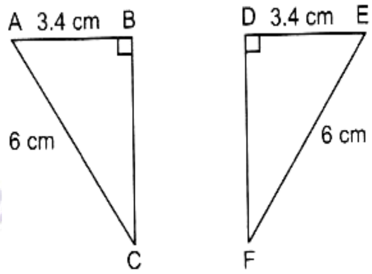
Sol : The sum of any two sides of a triangle is greater than the third side,

As (1.5cm + 3.4 cm = 4.9cm) is not greater than 5cm, so the length of third side of

Triangle cannot be 3.4cm,

$\therefore$ Correct option is (d)

7. In two right – angled ΔABC and ΔDEF, the measurement of hypotenuse and one side is given. Check if they are congruent or not? If yes, state the rule.



Sol : yes, ΔABC $\cong \Delta EDF$

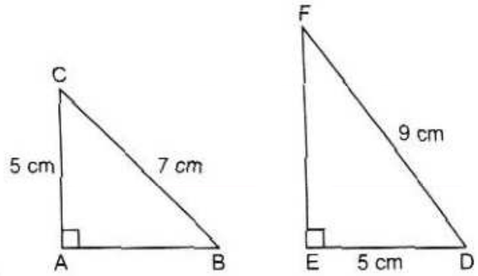
BY RHS Congruence rule.

RHS Congruence rule: If in two right angled triangles the hypotenuse and one side of one triangle are equal to the hypotenuse and one side of the other triangle, then the two triangles are congruent.

8. Examine the congruence of two triangles, whose measurements of some parts are given below:

(i) for ΔABC, $\angle A = 90^0, AC = 5cm, BC = 7cm$

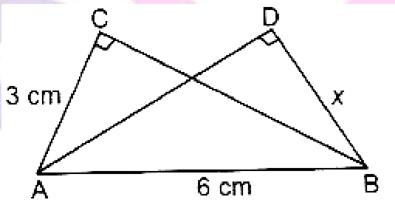
(ii) for ΔDEF, $\angle E = 90^0, DF = 9cm, DE = 5cm$



Sol : From the figure, $AC = DE = 5cm$, $\angle A = \angle E = 90^0$ but $BC \neq DF$

Hence, the given triangles are not congruent,

9. ΔACB and ΔADB are two congruent right- angled triangles on the same base, AB (=6cm) as shown in figure. If AC=3cm, find BD.



Sol. $\Delta ABC \cong \Delta ABD$

[RHS congruence rule given]

$\Rightarrow$ $AC = BD$ [CPCT]

$\Rightarrow$ $BD = 3cm$ $(\because AC = 3\ cm\ given)$

10. Fill in the blanks

(i) If two angles of a triangle are unequal then the smaller angle has the_________side opposite to it

(ii) The sum of any two sides of a triangle is __________ than the third side.

Sol : (i) Smaller (ii) Greater

True or False

11. Which of the following statements are true and which are false?

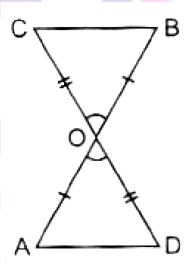
(i) If two sides of a triangle are unequal. Then longer side has the smaller angle opposite to it.

(ii) The sum of the three sides of a triangle is less than the sum of its three altitudes.

Sol : (i) false (ii) false.

I Short Answer Question

1. In The given figure, if OA = OB, OD = OC. Prove that $\Delta AOD \cong \Delta BOC$



Given : (i) OA = OB (ii) OD = DC

To prove :$\Delta AOD \cong \Delta BOC$

Proof : In ΔAOD and ΔBOC,

OA = OB (Given)

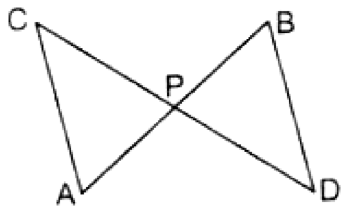
$\angle AOD = \angle BOC$ (Vertically opposite angles)

OD = DC (Given)

$\Rightarrow \Delta AOD = \Delta BOC$ (SAS congruence rule)

Hence proved

2. In the given figure AC = BD and AC ∥ DB. Prove that Δ APC ≅ Δ BPD



Proof : Given AC ∥ DB

AB is transversal

$\Rightarrow \angle PAC = \angle PBD$

(Alternate interior angles)

When CD is transversal, then

$\angle PAC = \angle PDB$

(Alternate interior angles)

Now, in ΔAPC and ΔBPD,

$\angle A = \angle B$ (As proved above)

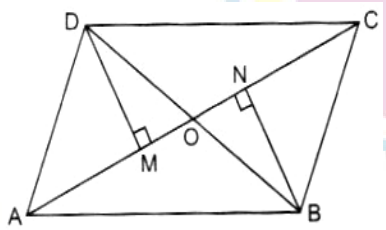
$AC = BD$ (Given)

$\angle C = \angle D$ (As proved above)

$\Rightarrow \Delta APC \cong \Delta BPD$ (ASA congruence rule)

Hence proved

3. In quadrilateral ABCD, BN and DM are drawn perpendicular to AC. Such that BN = DM. Prove that O is mid-point of BD.



Sol: In ΔDMO and ΔBNO,

$\angle DMO = \angle BNO = 90^0$ (Given)

$\angle DMO = \angle BNO$

(Vertically opposite angles)

DM = BN (Given)

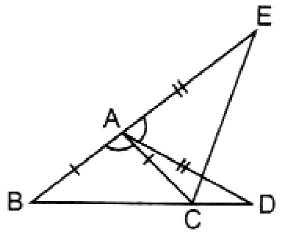
$\therefore \Delta DMO \cong \Delta BNO$ (AAS congruence rule)

$\Rightarrow$ DO = BO (CPCT)

$\Rightarrow$ 0 is mid – point of BD Hence proved

II Short Answer Questions

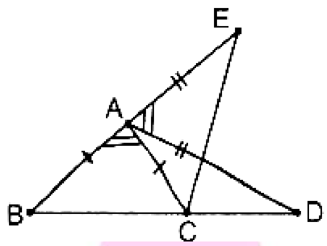
1. In the given figure, AB=AC, AD = AE and $\angle BAC = \angle DAE$, Prove that $\Delta BAD \cong \Delta CAE$.
[CBSE2010]



Sol : Given AB = AC

AD = AE

And $\angle BAC = \angle DAE$



To prove : $\angle BAD = \angle CAE$

Proof : $\angle BAC = \angle DAE$ (Given)

On adding $\angle DAC$ both sides, we get

$\angle BAC + \angle DAC = \angle DAE = \angle DAC$

$\Rightarrow \angle BAD = \angle EAC$

In ΔBAD and ΔEAC

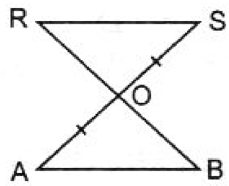
BA = CA (Given)

$\angle BAD = \angle EAC$ (Proved above)

AD = AE (Given)

$\Delta BAD \cong \Delta EAC$ (SAS congruence rule)

2. In the given figure, the line segment AB is parallel to another line segment RS and O is the mid-point of AS. Show that .



(i) ΔAOB ≅ ΔSOR

(ii) O is mid-point of BR **[CBSE 2013]**

Sol : Given (i) AB ∥ RS

(ii) O is mid-point of AS

To prove: (i) ΔAOB ≅ ΔSOR

(ii) O is mid-point of BR

Proof :

(i) Given : AB ∥ RS and AS is transversal

⟹ ∠OAB = ∠OSR (Alternate interior angles)

Now in ΔAOB and ΔSOR

∠OAB = ∠OSR (Proved above)

OA = OS [O is mid-point of AS]

And ∠OAB = ∠SOR

[Vertically opposite angles]

⟹ ΔAOB ≅ ΔSOR (ASA congruence rule)

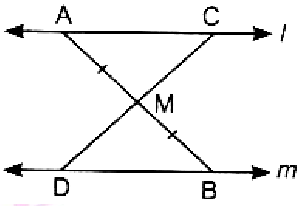
Hence Proved

(ii) As ΔAOB ≅ ΔSOR

So OB = OR (CPCT)

⟹O is mid-point of BR

3. In the given figure $l \| m$ and M is the mid-point of line segment AB. Prove that M is also the mid-point of any line segment CD having its end points C and D on l and m respectively .



Sol : Given (i) $l \parallel m$

(ii) M is mid-point of AB

To prove : M is mid-point of CD

Proof : Given $l \parallel m$ and AB is transversal

$\Rightarrow \angle CAM = \angle DBM$ (Alternate interior angles)

Now in ΔAMC and ΔBMD

$\angle CAM = \angle DBM$ (Proved above)

$AM = BM$ [M is mid-point of AB]

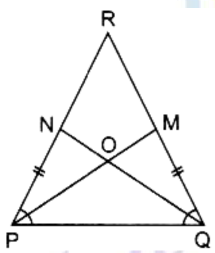
$\angle AMC = \angle BMD$ [Vertically opposite angles]

$\Rightarrow \Delta AMC \cong \Delta BMD$ (ASA congruence rule)

$\Rightarrow CM = DM$ [CPCT]

$\Rightarrow$ M is mid-point of CD. Hence Proved

4. In the given figure, $\angle QPR = \angle PQR$, and M and N are respectively points on the sides QR and PR of ΔPQR such that $QM = PN$, Prove that $OP = OQ$, where O is the point of intersection of PM and QN



Sol : Given $\angle PQR = \angle QPR$, M and N are two points on QR and PR such that $QM = PN$, PM and QN intersect at O.

To prove : $OP = OQ$

Proof : In ΔPQM and ΔPQN, we have

$QM = PN$ [Given]

$\therefore \angle QPM = \angle PQN$ [Given : $\angle QPR = \angle PQR$]

$PQ = PQ$ [Common]

$\therefore \Delta PQM = \Delta PQN$ (SAS congruence rule)

$\therefore \angle QPM = \angle PQN$

But $\angle QPN = \angle PQM$

$\Rightarrow \angle QPN - \angle QPM = \angle PQM - \angle PQM$

$\Rightarrow \angle OPN - \angle OQM$

Again in ΔPON and ΔQOM, we have

$PN = QM$ [Given]

$\therefore \angle OPN = \angle OQM$ [As proved]

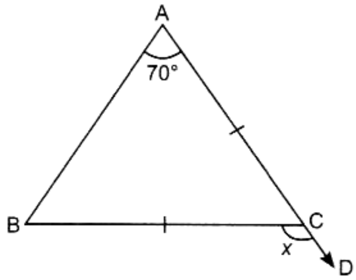
$\angle PON = \angle QOM$ [Vertically opposite angles]

$\Delta PON \cong \Delta QOM$ [AAS congruence rule]

$OP = OQ$ Hence Proved.

III Short Answer Questions

1. In the given figure, ΔABC is an isosceles triangle with AC = BC. Find the value of x



Sol : angles opposite to equal sides are equal. As $AC = BC$ in ΔABC

$\Rightarrow \angle B = \angle A = 70^0$

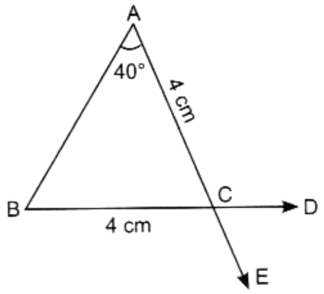
Now,

$\therefore \angle BCD = \angle A + \angle B$ [By exterior angle theorem]

$\Rightarrow x = 70^0 + 70^0 = 140°$

2. In the given figure, AC = BC = 4 cm and ∠A = 40^0, then find ∠DCE.

Sol : Angles opposite to equal sides are equal



$\therefore \angle A = \angle B = 40^0$,

Now, $\angle A + \angle B + \angle ACB = 180^0$,

(Angle sum property of a triangle)

$\Rightarrow 40^0 + 40^0 + \angle ACB = 180^0$

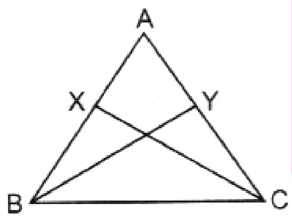
$\Rightarrow \angle ACB = 180^0 - 80^0 = 100^0$

$\therefore \angle DCE = \angle ACB$

(Vertically opposite angles)

$\Rightarrow \angle DCE = 100^0$

3. In the figure below, ABC is a triangle in which AB = AC, X and Y are points on AB and AC such that AX = AY. Prove that ΔABY ≅ ΔACX.



Given : In Δ ABC, AB = AC and AX = AY

To prove : ΔABC≅ ΔACX

Proof : ΔABC andΔACX

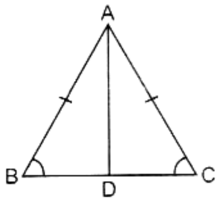
AB = AC (Given)

∠A = ∠A (Common)

AX = AY (Given)

⟹ Δ ABY ≅ Δ ACX (SAS congruence rule)

4. In the given figure ΔABC is an isosceles triangle with AB = AC. If the altitude is drawn from one of its vertex, then prove that it bisects the opposite side.



Given : (i) Δ ABC is an isosceles triangle with AB = AC

(ii) AD is the altitude is drawn from vertex A, on side BC

To Prove: D is mid – point of BC i.e., BD = CD

Proof : In Δ ABD and Δ ACD

AB = AC (Given)

$\angle B = \angle C$ (Angles opposite to equal sides are equal)

$\angle ADB = \angle ADC = 90^0$ (Given)

$\Delta ABD \cong \Delta ACD$ (AAS congruence rule)

BD = CD (CPCT)

Therefore, AD bisect BC.

Hence proved.

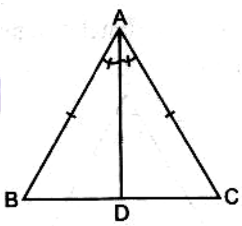
IV Short Answer Questions

5. Prove that angels opposite to equal sides of an isosceles triangle are equal.

Given: ΔABC is an isosceles triangle with AB = AC

To prove : $\angle B = \angle C$

Construction : Draw AD bisector of $\angle A$ which intersects BC at D.



Proof: In ΔBAD and ΔCAD

AB = AC (Given)

$\angle BAD = \angle CAD$ (By construction)

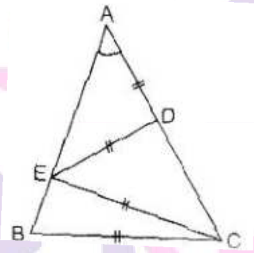
$AD = AD$ (Common)

So, $\Delta BAD \cong \Delta CAD$ (SAS congruence rule)

$\Rightarrow \angle ABD = \angle ACD$ (CPCT)

So, $\angle B = \angle C$ Hence proved

6. In the given figure AB = AC, D is point on AC and E on AB such that AD = ED = EC = BC. Prove that $\angle A : \angle B = 1 : 3$



Given : (i) $AB = AC$

(ii) $AD = ED = EC = BC$

To prove : $\angle A : \angle B = 1:3$

Proof : In ΔAED,

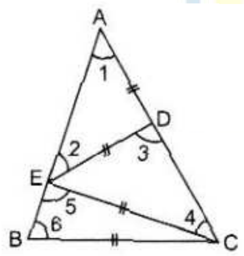
$AD = ED$ (Given)

$\Rightarrow \angle 1 = \angle 2$ -----(1)

(Angles opposite to equal sides are equal)

Also in $\angle AED$, $\angle A + \angle AED + \angle ADE = 180^0$

(Angle sum property of triangle)



$\Rightarrow \angle 1 + \angle 2 + \angle ADE = 180^0$

$\Rightarrow \angle ADE = 180^0 - 2\angle 1$ ($\angle 1 = \angle 2$)

But $\angle ADE + \angle CDE = 180^0$

(Linear pair axiom)

$\Rightarrow 180^0 - 2\angle 1 + \angle 3 = 180°$

$\Rightarrow \angle 3 = 2\angle 1$ ----(ii)

Now in Δ CDE,

$\angle 3 + \angle CED + \angle 4 = 180^0$

(Angle sum property of triangle)

$\Rightarrow \angle CED = 180^0 - \angle 3 - \angle 4$

$\angle CED = 180^0 - 2\angle 3$ ----(iii)

($\because$ ED = EC; $\angle 3 = \angle 4$)

Again, $\angle AED + \angle CED + \angle BEC = 180^0$

(Linear pair axiom)

$\Rightarrow \angle 2 + 180^0 - 2\angle 3 + \angle 5 = 180^0$

$\Rightarrow 2\angle 3 = \angle 2 + \angle 5$ -----(iv)

In ΔBEC, EC = BC

$\Rightarrow \angle 6 = \angle 5$ -----(v)

(Angles opposite to equal sides are equal)

From (i), (iv) and (v), we get

$2\angle 3 = \angle 1 + \angle 6$

($\angle 2 = \angle 1$ and $\angle 5 = \angle 6$)

$\Rightarrow 2(2\angle 1) = \angle 1 + \angle 6$ [From (ii)]

$\Rightarrow 4\angle 1 = \angle 1 + \angle 6$

$\Rightarrow \angle 6 = 3\angle 1$

$\Rightarrow \angle B = 3\angle A$

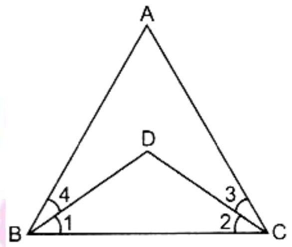
$\therefore \frac{\angle A}{\angle B} = \frac{1}{3}$

$\Rightarrow \angle A : \angle B = 1:3$ Hence proved

7. **In the given figure, we have $\angle ABC = \angle ACB$ and $\angle 3 = \angle 4$. Show that**

i) $\angle 1 = \angle 2$

ii) Justify which two sides of Δ ABC are equal.



i) Given $\angle ABC = \angle ACB$

$\Rightarrow \angle 1, + \angle 4 = \angle 2 + \angle 3$

But $\angle 3 = \angle 4$ (Given)

$\Rightarrow \angle 1 = \angle 2$ Hence proved

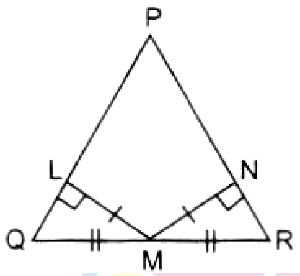
ii) $\angle ABC = \angle ACB$ (Given)

$\Rightarrow AC = AB$

Because it an isosceles triangle, the sides opposite to equal angles are equal.

V Short Answer Questions

1. **In the given figure, LM = MN, QM = MR ML $\perp$ PQ and MN $\perp$ PR. Prove that PQ = PR.**



Sol. Given : LM = MN, QM = MR

ML $\perp$ PQ and MN $\perp$ PR

To prove: PQ = PR

Proof : In ΔQML and ΔRMN,

LM = MN [Given]

$\angle L = \angle N$ [Each 90^0]

$QM = MR$ [Given]

$\Rightarrow \Delta QML = \Delta RMN$, [RHS congruence rule]

$\Rightarrow \angle LQM = \angle NRM$ [CPCT]

$\Rightarrow PQ = PR$

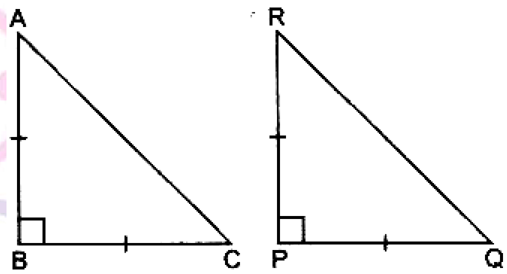
[Sides opposite to equal angles are equal]

Hence proved.

2. What additional information is needed for establishing $\Delta ABC \cong \Delta RPQ$, by RHS congruence rule, if it is given that AB = RP and $\angle B = \angle P = 90^0$?

Sol. Given : $\Delta ABC \cong \Delta RPQ$

$AB = RP$



$\angle B = \angle P = 90^0$

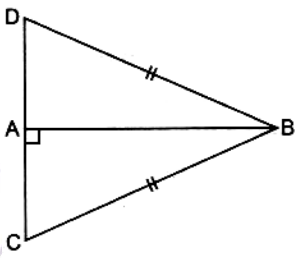
$\Rightarrow A \leftrightarrow R$

$B \leftrightarrow P$ and $C \leftrightarrow Q$

So, for congruence of ΔABC and ΔRPQ by RHS congruence rule, we must have

$AC = RQ$

3. Write the congruence statement by the information shown in the figure.



Sol. From the figure:

$In\ \Delta BAC = \Delta BAD$,

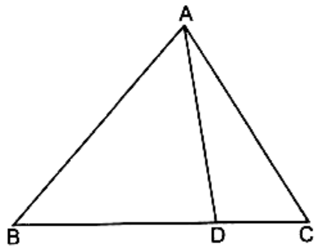
AB = AB [Common]

$\angle BAC = \angle BAD$ [Each 90^0]

BC = BD [Given]

$\Rightarrow \Delta BAC \cong \Delta BAD$ [RHS congruence rule]

4. In the given figure, AB > AC and D is any point on sie BC of ΔABC. Prove that AB > AD.



Sol : AB > AC [Given]

$\angle C > \angle B$

[Angle opposite to longer side is larger] ……(i)

Now, $\angle ADB$ is the exterior angle of ΔADC

$\Rightarrow \angle ADB = \angle DAC + \angle C$

$\Rightarrow \angle ADB > \angle C$ ….(ii)

Therefore, from (i), we get

$\Rightarrow \angle ADB > \angle B$

Now in ΔABD

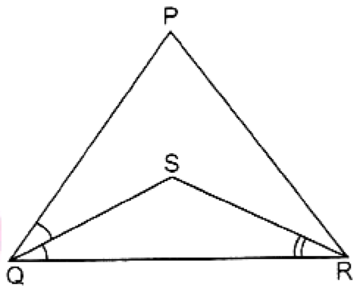
$\angle ADB > \angle B$

AB > AD

[Side opposite to greater angle is longer]

Hence proved.

5. **In the given figure, PQ > PR, QS and RS are the bisectors of ∠Q and ∠R respectively. Prove that SQ > SR.**



Sol : Proof : In ΔPQR

PQ > PR [Given]

$\Rightarrow$ ∠PRQ > $\angle PQR$ (Angle opposite to long side is larger)

$\Rightarrow \frac{1}{2}\angle PRQ > \frac{1}{2}\angle PQR$

$\Rightarrow$ ∠SRQ > $\angle SQR$ [Givne QS and RS are the bisectors of ∠Q and ∠R respectively]

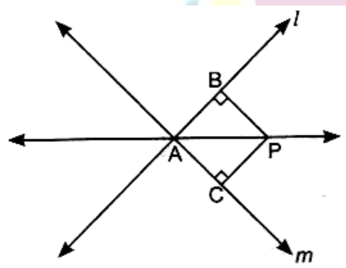
SQ > SR

(Side opposite to greater angle is larger)

Hence Proved.

VI Short Answer Questions

1. **P is a point equidistant from two lines l and m intersecting at point A as shown in figure. Show that line AP bisects the angle between them.**



Sol : Given: (i) Lines l and m intersect each other at point A

(ii) From figure PB ⊥ l, PC ⊥ m

(iii) PB = PC

To prove : Line AP bisects ∠BAC

Proof : In ΔPAB and ΔPAC

PB = PC [Given]

$\angle PBA = \angle PCA = 90^0$ [Given]

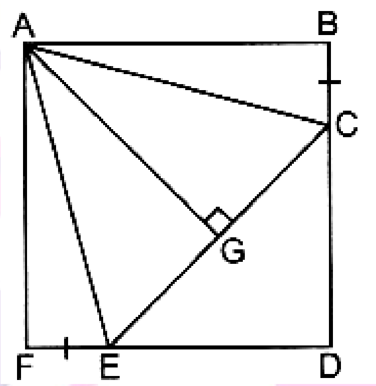
PA = PA [Common]

$\Rightarrow \Delta PAB \cong \Delta PAC$ [RHS congruence rule]

$\Rightarrow \angle PAB = \angle PAC$ [CPCT]

$\Rightarrow$ *Line AP bisects* $\angle BAC$. Hence proved.

2. ABDF is a square and BC = EF in the given figure, Prove that



(i) ΔABC $\cong$ ΔAFE (ii) ΔACG $\cong$ ΔAEG [HOTS]

Given : (i) ABDF is square

(ii) BC = EF

To Prove: (i) $\Delta ABC \cong \Delta AFE$

(ii) $\Delta ACG \cong \Delta AEG$

Proof :

(i) In $\Delta ABC \cong \Delta AFE$

AB = AF [All sides of square are equal

BC = FE [Given]

And $\angle ABC = \angle AFE = 90°$

[Each angle of a square is a right angle]

$\Rightarrow \Delta ABC \cong \Delta AFE$ [SAS congruence rule]

$\Rightarrow$ $AC = AE$ [CPCT]

Hence Proved.

(ii) ΔACG *and* ΔAEG

AC = AE [Proved above]

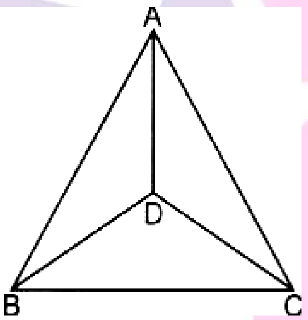
AG = AG [Common]

$\angle AGC = \angle AGE = 90^0$ [Given]

$\Rightarrow \Delta ACG \cong \Delta AEG$ [RHS congruence rule]

Hence proved.

3. In the given figure, AB = AC and D is a point in the interior of ΔABC such that $\angle$DBC = $\angle$DCB. Prove that AD bisects $\angle$BAC of ΔABC



Sol : In ΔBDC

$\angle DBC = \angle DCB$ [Given]

$\Rightarrow$ BD = CD [Sides opposite to equal angles are equal]

Now, in ΔABD and ΔACD,

AB = AC [Given]

BD = CD [Proved above]

And AD = AD [Common]

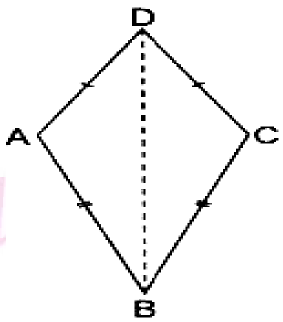
$\Rightarrow$ $\Delta ABD \cong \Delta ACD$ [SSS congruence rule]

$\angle BAD = \angle CAD$ [CPCT]

Hence, AD bisects $\angle$BAC,

Hence Proved

4. In the given figure, AD = CD and AB = CB. Prove that



(i) $\Delta ABD \cong \Delta CBD$

(ii) **BD bisects $\angle ABC$**

Sol : **Given** AD = CD and AB = CD

To prove

(i) $\Delta ABD \cong \Delta CBD$

(ii) $\angle ABD = \angle CBD$, i.e., BD bisects $\angle ABC$

Proof :

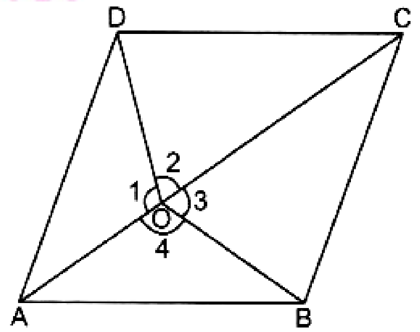
(i) In ΔABD and ΔCBD

	AB = CB	[Given]
	AD = CD	[Given]
	BD = BD	[Common]
$\Rightarrow$	$\Delta ABD \cong \Delta CBD$	[SSS congruence rule]
(ii) Since	$\Delta ABD \cong \Delta CBD$	[Proved above]
$\Rightarrow$	$\angle ABD = \angle CBD$	[CPCT]
$\Rightarrow$	BD bisects $\angle ABC$,	Hence Proved

5. A point O is taken inside an equilateral four sided figure ABCD such that its distances from the angular points D and B are equal. Show that AO and OC are together form one and the same straight line.

Given: O is a point anywhere inside an equilateral four sided figure ABCD such that OD = OB.

To prove : AO and OC are in the same straight line



Proof : In Δ AOD and Δ BOA,

AD = AB

(Given sides of ABCD are equal)

A0 = AO (common)

OD = OB (Given)

$\Rightarrow \Delta AOD \cong \Delta BOA$, (SSS congruence rule)

$\Rightarrow \angle 1 = \angle 4$ [CPCT]

Similarly, in ΔCOD and ΔCOB,

CO = CO [Common]

CD = CB [Given sides of ABCD are equal]

OD = OB [Given]

$\Rightarrow \Delta COD \cong \Delta COB$, (SSS congruence rule)

$\Rightarrow \angle 2 = \angle 3$ [CPCT]

But, $\angle 1 + \angle 2 + \angle 3 + \angle 4 = 360^0$ [Complete Angle]

$\Rightarrow \angle 1 + \angle 2 + \angle 2 + \angle 1 = 360^0$

$[\because \angle 4 = \angle 1 \; and \; \angle 3 = \angle 2]$

$\Rightarrow 2(\angle 1 + \angle 2) = 360^0$

$\Rightarrow \angle 1 + \angle 2 = 180^0$

$\Rightarrow \angle AOD + \angle COD = 180^0$

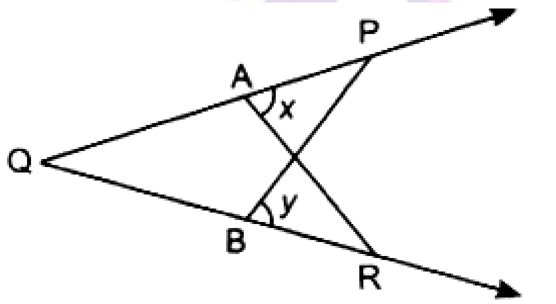
But these are the linear pair angles formed by a line OD stands on AOC

Therefore, AO and OC are together form one and the same straight line

$\Rightarrow$ AOC is a traight line. Hence proved.

I Long Answer Questions

1. In the given figure, PQ = QR and $\angle x = \angle y$. Prove that AR = PB.



Proof : In the figure.

$\angle QAR + \angle PAR = 180^0$ (Linear pair axiom)

$\Rightarrow \angle QAR + \angle x = 180^0$

$\Rightarrow \angle QAR = 180^0 - \angle x^0$ ----- (i)

Similarly, $\angle QBP + \angle RBP = 180^0$ (Linear pair axiom)

$\Rightarrow \angle QBP + \angle y = 180^0$

$\Rightarrow \angle QBP = 180^0 - \angle y$ ----- (ii)

But Given, $\angle x = \angle y$

$\therefore \angle QAR = \angle QBP$ [From (i) and (ii)]

Now, in ΔQAR and ΔQBP,

$QR = PQ$ (Given)

$\angle QAR = \angle QBP$ [As proved above]

$\Rightarrow Q = \angle Q$ (Common)

$\Rightarrow \Delta QAR \cong \Delta QBP$ (AAS congruence rule)

$\Rightarrow AR = PB$ (CPCT)

Hence proved.

2. Prove that "Two triangles are congruent, if two angles and the included side of one triangle are equal to two angles and the included side of other triangle".

Given : two triangles ABC and PQR in which

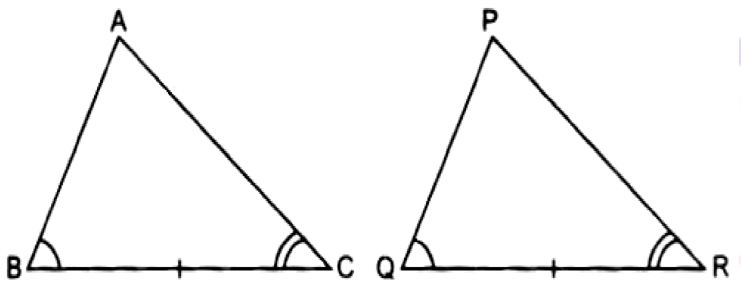
$\angle B = \angle Q$, $\angle C = \angle R$

And $BC = QR$

To prove : $\Delta ABC \cong \Delta PQR$

Proof : Three cases arises

Case 1 : When $AB = PQ$, $\angle B = \angle Q$ and $BC = OR$



In ΔABC and ΔPQR,

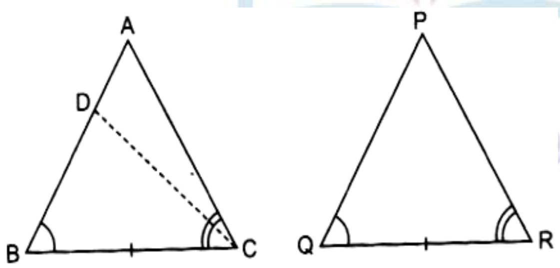
$AB = PQ$ (Assumed)

$\angle B = \angle Q$ (Given)

$BC = QR$ (Given)

$\Rightarrow \Delta ABC \cong \Delta PQR$ (SAS congruence rule)

Case II. When $AB > PQ$



Let us consider a point D on AB such that $DB = PQ$ Now, consider ΔDBC and ΔPQR

$DB = PQ$ (By construction)

$\angle B = \angle Q$ (Given)

$BC = QR$ (Given)

$\Rightarrow \Delta DBC \cong \Delta PQR$ (SAS congruence rule)

$\Rightarrow \angle DCB = \angle PRQ$ (CPCT)

But, we are given that

$\angle ACB = \angle PRQ$

So, $\angle ACB = \angle DCB$

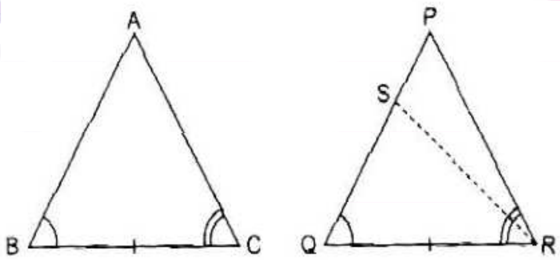
This is possible only when D coincides with A

i.e. $BA = QP$

So, $\Delta ABC \cong \Delta PQR$ (SAS congruence rule)

Case III. When $AB < PQ$

Let us consider a point S on PQ such that $SQ = AB$ as shown in figure



Now, consider ΔABC and ΔSQR

$AB = SQ$ (By construction)

$\angle B = \angle Q$

$BC = QR$

So, $\Delta ABC \cong \Delta SQR$ (SAS congruence rule)

$\Rightarrow \angle ACB = \angle SRQ$ (CPCT)

But, we are given that

$\angle ACB = \angle PRQ$ (As $\Delta ABC \cong \Delta PQR$)

$\Rightarrow \angle SRQ = \angle PRQ$

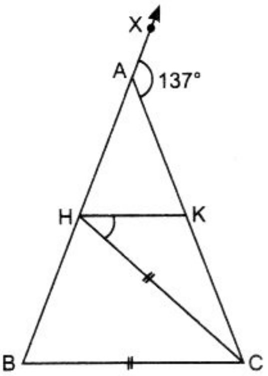
This is possible only when S coincide with P

Or $QS = QP$

So, $\Delta ABC \cong \Delta PQR$ Hence proved.

II Long Answer Questions

1. In the given figure, AB = AC, CH = CB and HK ∥ BC. If ∠CAX = 137^0, then find ∠CHK



Given : In ΔABC,

(i) $AB = AC$

(ii) $CH = CB$

(iii) $HK \parallel BC$

(iv) $\angle CAX = 137^0$

To find : $\angle CHK$

Finding : In ΔABC, $AB = BC$ (Given)

$\Rightarrow \angle ABC = \angle ACB$

(Angles opposite to equal sides are equal)

But $\angle CAX = \angle ABC + \angle ACB$

(By exterior angle theorem)

$\Rightarrow 137^0 = 2\angle ABC$ $(\because \angle ACB = \angle ABC)$

$= \angle ABC = \frac{137^0}{2} = 68.5^0$

$\Rightarrow \angle ACB = 68.5^0$

Now, CH = CB

$\Rightarrow \angle CBH = \angle CHB$

(Angles opposite to equal sides are equal)

$\Rightarrow \angle CHB = 68.5^0$ ($\angle CBH = \angle ABC$)

Again HK ∥ BC (Given)

and CH is transversal

$\Rightarrow \angle BHK + \angle CBH = 180^0$ (Co- interior angles)

$\Rightarrow \angle CHB + \angle CHK + \angle CBH = 180^0$ ($\because \angle BHK = \angle CHB + \angle CHK$)

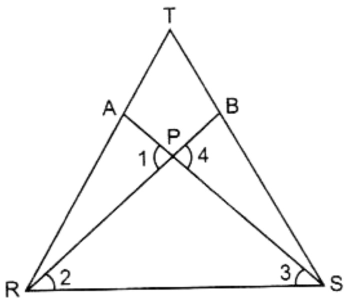
$2 \angle CHB + \angle CHK = 180^0$ ($\angle CBH = \angle CHB$)

$\Rightarrow 2 \times 68.5^0 + \angle CHK = 180^0$

$\Rightarrow \angle CHK = 180^0 - 137^0 = 43^0$

2. In the given figure , it is given that RT = TS ,

$\angle 1 = 2\angle 2$ and $\angle 4 = 2\angle 3$.



Prove that Δ RBT $\cong \Delta$ SAT

Given i) RT = TS

ii) $\angle 1 = 2\angle 2$

iii) $\angle 4 = 2\angle 3$

To prove Δ RBT $\cong \Delta$ SAT

Proof : In Δ TRS

RT = TS (Given)

$\Rightarrow \angle TRS = \angle TSR$

(Angles opposite to equal sides are equal) --- (i)

Now, SA and RB intersect at a point. Let it be P.

So, $\angle 1 = \angle 4$ (Vertically opposite angles)

$\Rightarrow 2\angle 2 = 2\angle 3$

$\Rightarrow \angle 2 = \angle 3$ ----(ii)

Now, in Δ RPS,

$\angle 2 = \angle 3$ (Proved above)

$\Rightarrow SP = RP$ (Sides opposite to equal angles are equal) -----(iii)

Again from (i),

$\angle TRS = \angle TSR$

$\Rightarrow \angle ARP + \angle 2 = \angle BSP + \angle 3$

$\Rightarrow \angle ARP = \angle BSP$ (As $\angle 2 = \angle 3$) ----- (iv)

Now in ΔARP and Δ BSP,

$\angle ARP = \angle BSP$ (From (iv))

$RP = SP$ (From (iii))

$\angle 1 = \angle 4$ (Vertically opposite angles)

$\Rightarrow \Delta\ ARP \cong \Delta\ BSP$, (ASA congruence rule)

$\Rightarrow$ $AR = BS$ (CPCT)

But $RT = TS$ (Given)

$\Rightarrow RT - AR = TS - BS$

$\Rightarrow AT = BT$ --- (iv)

Now, in ΔRBT and ΔSAT

$RT = ST$ (Given)

$\angle T = \angle T$ (Common)

$BT = AT$ (From (v))

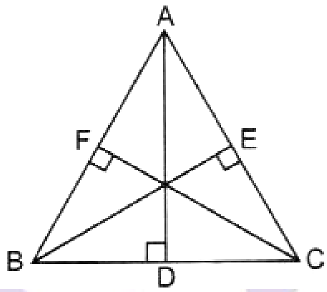
$\Rightarrow \Delta\ RBT \cong \Delta\ SAT$, (SAS congruence rule)

Hence proved

III Long Answer Questions

1. **Prove that the sum of three altitudes of a triangle is less than the sum of the three sides of a triangle,** **[HOTS]**

Sol : Given : In ABC, AD, BE and CF are the altitudes on sides BC, CA and AB respectively.



To prove : $AD + BE + CF < AB + BC + CA$

Proof : Since perpendicular line segment is the shortest line segment, then

When $AD \perp BC$ we have $AB > AD$ and $AC > AD$

$\Rightarrow AB + AC > AD + AD$

$\Rightarrow AB + AC > 2AD$-----(i)

Similarly, when $BE \perp AC$, then

$BA + BC > 2BE$ ------(ii)

and , when $CF \perp AB$ $CA + CB > 2CF$ ----(iii)

Adding (i), (ii) and (iii) , we get

$AB + AC + BA + BC + CA + CB > 2AD + 2BE + 2CF$

$\Rightarrow 2AB + 2BC + 2CA > 2AD + 2BE + 2CF$

$\Rightarrow 2(AB + BC + CA) > 2(AD + BE + CF)$

$\Rightarrow AB + BC + CA > AD + BE + CF$

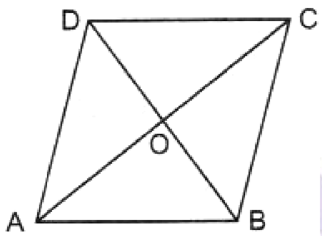
$AD + BE + CF < AB + BC + CA$ Hence proved.

2. Diagonal AC and BD of quadrilateral ABCD intersects each other at O. Prove that

i) AB + BC + CD + DA > AC + BD

ii) AB + BC + CD + DA < 2(AC + BD)

Given : AC and BD are the diagonals of quadrilateral ABCD.



i) **To prove :** AB + BC + CD + DA > AC + BD

Proof : We know that the sum of any two sides of a triangle is always greater than the third side. Therefore,

In Δ ABC, AB + BC> AC -----(i)

In Δ BCD, BC + CD> BD -----(ii)

In Δ CDA, CD + DA> CA -----(iii)

In Δ ABD, AB + AD> BD -----(iv)

Adding (i), (ii), (iii) and (iv) we get

2 (AB + BC + CD + DA)< 2 (AC + BD)

$\Rightarrow$ AB + BC + CD + DA > AC +BD

Hence proved

ii) **To prove :** AB + BC + CD + DA < 2(AC + BD)

Proof : In ΔOAB,

OA + OB > AB ----(i)

In ΔBOC, OB + OC > BC ----(ii)

In ΔCOD, OC + OD > CD ----(iii)

In ΔAOD, OA + OD > DA ----(iv)

Adding (i), (ii), (iii) and (iv), we get

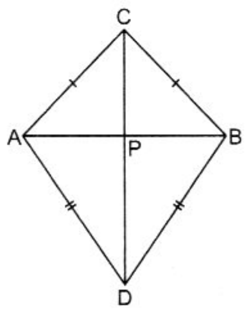
2 (OA + OB + OC + OD)> AB + BC + CD + DA

$2[OA + OC) + (OB + OD) > AB + BC + CD + DA$

$2(AC + BD) > AB + BC + CD + DA$

$AB + BC + CD + DA < 2(AC + BD)$ Hence proved

3. AB is a line segment C and D are points on opposite sides of AB such that each of them is equidistant from the point A and B as shown in figure. Show that the line CD is the perpendicular bisector of AB.



Given : $CA = CB$ and $DA = DB$

To prove : (i) $CD \perp AB$

(ii) CD bisects AB

Proof : Let CD intersects AB at P,

Consider ΔCAD and ΔCBD

$CA = CB$ (Given)

$DA = DB$ (Given)

$CD = CD$ (Common)

$\Rightarrow \Delta CAD \cong \Delta CBD$ (SSS congruence rule)

$\Rightarrow \angle ACD = \angle BCD$ (CPCT)

Again, in ΔCAP and ΔCBP

$CA = CB$ (Given)

$CP = CP$ (Common)

$\angle ACP = \angle BCP$ (Proved above)

$\Rightarrow \Delta CAP \cong \Delta CBP$ (SAS congruence rule)

$\Rightarrow AP = BP$ (CPCT)

$\angle APC = \angle BPC$ (CPCT)

But, these are the linear pair angles.

Therefore, $\angle APC = \angle BPC = 180^0$

$\Rightarrow 2\angle APC = 180^0$

$\Rightarrow \angle APC = 90^0$

$\Rightarrow CD \perp AB$

Hence $AP = BP$ and $\angle APC = 90^0$. This indicates that CD is perpendicular bisector of AB.

Hence Proved.

4. Prove that two right triangles are congruent, if the hypotenuse and a side of one triangle are respectively equal to the hypotenuse and a side of the other triangle. [HOTS].

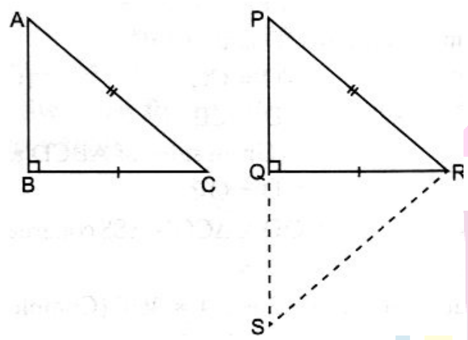
Sol : Given

(i) ΔABC and ΔPQR are the are the two right angled triangles with $\angle B = 90^0$ and $\angle Q = 90^0$

(ii) $AC = PR$ and $BC = QR$

To prove: $\Delta ABC \cong \Delta PQR$

Construction: Produce PQ To S such that $QS = AB$ Join S and R.



Proof : In ΔABC and ΔSQR, we have

$AB = SQ$ (By Construction)

$BC = QR$ (Given)

$\angle ABC = \angle SQR$ (Each 90^0)

$\Rightarrow \Delta ABC \cong \Delta SQR$ (SAS congruence rule)

$\Rightarrow \angle A = \angle S$ (CPCT) and $AC = SR$ (CPCT)

But $AC = PR$ (Given)

$\Rightarrow$ $SR = PR$

$\Rightarrow$ $\angle P = \angle S$

(Angles opposite to equal sides of ΔSPR are equal)

i.e. $\angle A = \angle P$

($\angle A = \angle S$ and $\angle S = \angle P$, Proved above)

Now, in ΔABC and ΔPQR

$\angle A = \angle P$ (Proved above)

$\angle B = \angle Q = 90^0$

$\angle C = \angle R$

$\therefore$ (By angle sum property of a triangle)

Again, in ΔABC and ΔPQR

$BC = QR$ (Given)

$AC = PR$ (Given)

$\angle C = \angle R$ (Proved above)

$\Rightarrow \Delta ABC \cong \Delta PQR$ (SAS congruence rule)

Hence proved