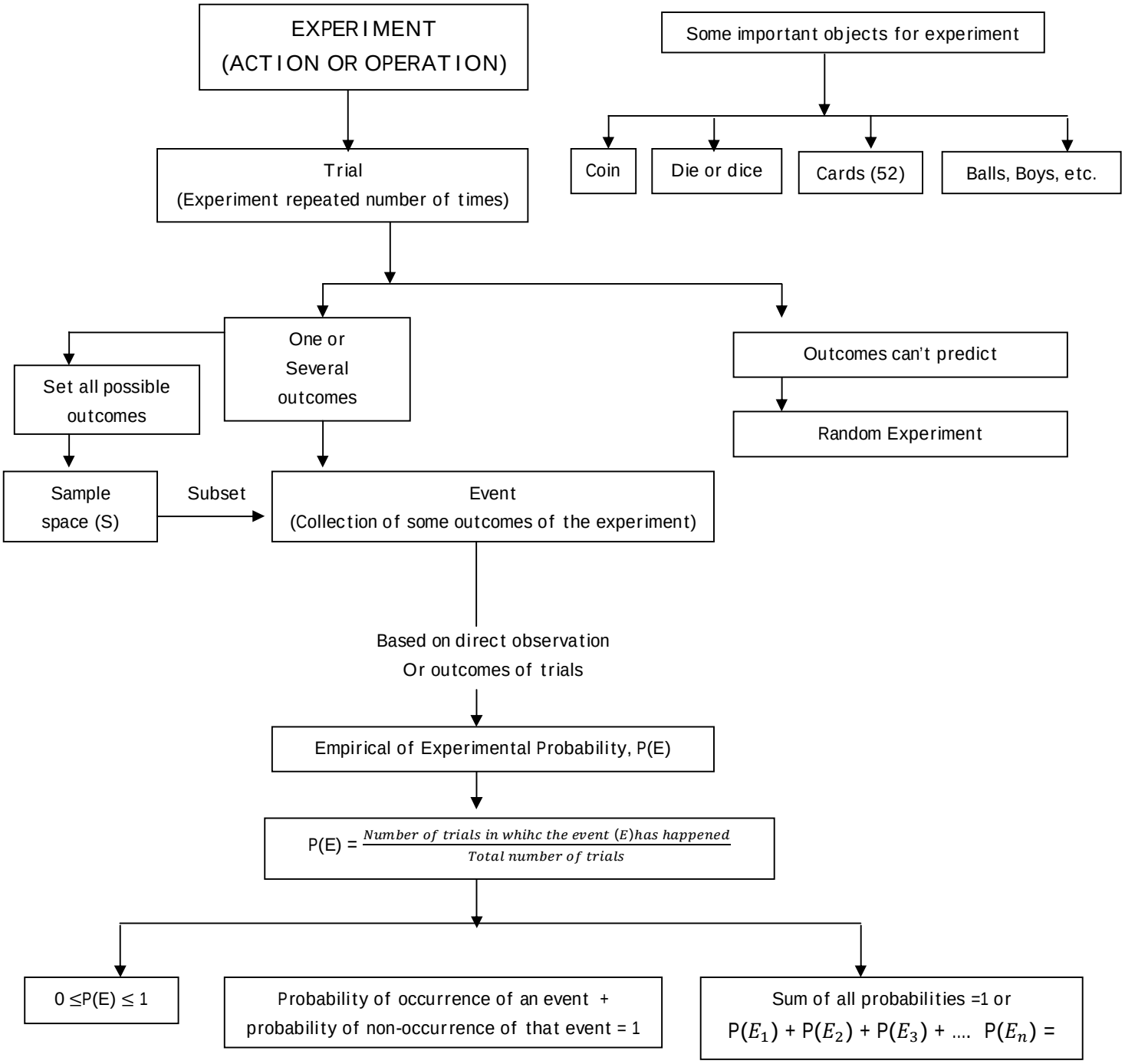
GRADE - 9

LESSON : 15 PROBABILITY

PROBABILITY – AN EXPERIMENTAL APPROAH



Know the terms

Number of Experiments

(i) When a coin is tossed

P(getting a head), $P(H) = \frac{Number\ of\ heads}{Total\ number\ of\ trials}$

And P (getting a tail), $P(T) = \frac{Number\ of\ tails}{Total\ number\ of\ trials}$

Also $P(H) + P(T) = 1$

i) When a die is tossed, $P(E) = \frac{Number\ Number\ of\ outcomes\ having\ a\ particular\ number\ of\ die}{Total\ number\ of\ times\ the\ die\ is\ rolled\ (thrown)}$

and $P(E_1) + P(E_2) + P(E_3) + P(E_4) + P(E_5) + P(E_6) = 1$

Where $P(E_1)$ = Probability of an event of getting outcome 1.

$P(E_2)$ = Probability of an events of getting outcome 2 and so on.

Note :

- In the similar way, one can find the probability of other experiments
- Probability of an event can be any fraction from 0 to 1

  P(E) + P (not E) = 1

- The empirical (or experimental) probability depends on the number of trials undertaken and the number of times the outcomes occurs in these trials.

Objective Type Questions

I. Multiple choice questions

1. Probability of an event can be

   a) -0.7 b) $\frac{11}{9}$ c) 1.001 d) 0.6

   Sol. (d) Probability of an event lies between 0 and 1 [both inclusive]

2. The probability of happening of an event is 45%. The probability of an event is

   a) 45 b) 4.5 c) 0.45 d) 0.045

   Sol. (c) 45% means 45 out of 100

   Therefore probability = $\frac{45}{100}$ = 0.45

3. In a class of 40 students, there are 110% girls. Then the number of girls is

   a) 44 b) 22 c) 30 d) None of these

   Sol. (d) Maximum is 100%. So 110% is not valid

4. Which of the following words represent uncertainty?

   a) Probability b) Value c) event d) None of these

   Sol. (a) Probability

5. Probability represents

   a) uncertainty b) certainty

   c) numerical measure of uncertainty d) numerical measure of certainty

   Sol. (c) numerical measure of uncertainty

6. In an experiment, probability of an event is better approximated, when an experiment is performed.

   a) 10 times b) 20 times c) 30 times d) large number of times

   Sol. (d) large number of times

7. Empiricalprobability of an event is also known as

a) an experimental probability b) a theoretical probability

c) theoretical expectation of a chance

Sol. a) an experimental probability

8. An experiment is performed and probability of an event A is recorded, probability of an event A can be

a) 0.001 b) 1.999 c) 1.001 d) -0.999

Sol. a) 0.001 because $0 \leq P(E) \leq 1$

9. An experiment is performed 350 times and there are three possible events A, B and C in an experiment. Possible occurrence of three events are recorded, which records are possible?

a) A : 166, B :80, C :94 b) A : 90, B :0, C :250

C) A : 200, B :100, C :50 d) A : 110, B :110, C :110

Sol. c) Here A = 200, B = 100, C= 50

As possible occurrence of three events are shown

Total possible outcomes

= A + B + C = 200 + 100 + 50 = 350

These are equal to number of trials

Or P (E) = $\frac{350}{350}$ = 1

Hence, this combination is possible.

10. In an experiment, the sum of probabilities of different events is

a) 1 b) 0.5 c) -2 d) $\frac{16}{15}$

Sol. a) 1

11. Write the formula for finding the empirical probability of an event

Sol. The empirical (or experimental) probability P(E) of an event E is given by

$$P(E) = \frac{\text{Number of trials in which the event (E) has happened}}{\text{Total number of trials}}$$

12. Find the probability of head coming up when a coin is tossed once

Sol. Total outcomes = 2

Number of head coming up =1

∴ Probability (head coming up) = $\frac{1}{2}$

13. Write the sample space of a coin tossed three times.

Sol. S(E) = {HHH, TTT, HHT, HTH, THH, TTH, THT, HTT}

∴ Total number of sample space = 8

14. A card is drawn at random from a well shuffled pack of 52 cards. Find the probability that the card drawn is a red card

Total number of red cards in a well shuffled packof 52 cards =26

∴ P (drawing a red card) $\frac{26}{52} = \frac{1}{2}$

I. Short answer type questions

15. There Are 13 girls and 15 boys in a line. If one student is chosen at random, then find the probability that he is a boy.

Sol. Total number of students = 13 + 15 = 28

Number of boys = 15

∴P (a boy) = $\frac{15}{28}$

16. 1000 families with 2 children were surveyed and the following data were recorded

Number of girls in a family	0	1	2
Number of families	111	614	275

If a family is chosen at random, compute the probability that it has :

i) exact 1 girl ii) exactly 2 girls

Sol. Total number of families = 1000

i) Number of families that have exactly one girl = 614

∴P (a family that has exactly one girl)

$= \frac{614}{1000} = 0.614$

(ii) Number of families that have exactly 2 girls = 275

∴P (a family that has exactly 2 girls)

$= \frac{275}{1000} = 0.275$

17. On One page of a telephone directory, there were 200 telephone numbers. The frequency distribution of their unit place digit is given in the following table

Digit	0	1	2	3	4	5	6	7	8	9
Frequency	22	26	22	22	20	10	14	28	16	20

What is the probability of a number chosen at random, that the digit in its unit place is 4?

Sol. P [digit 4 in the unit place of a telephone number]

$= \frac{Frequency\ of\ 4}{Total\ number\ of\ selected\ telephone\ numbers}$

$= \frac{20}{200} = \frac{1}{10} = 0.1$

18. If the probability of winning a game is 0.4. What is the probability of losing it?

[CBSE 2015]

Sol. We know that P(E) + P(not E) = 1

i.e. ,P(Winning a game) + P(losing a game) = 1

P(losing a game) = 1 - P(Winning a game)

= 1 – 0.4 = 0.6

Hence, probability of losing the game = 0.6

19. Two coins are tossed simultaneously 200 times and the following outcomes are recorded:

HH	HT/TH	TT
56	110	34

What is the empirical probability of occurrence of at least one head in the above case?

Sol. Total number of possible outcomes with at least one head = 56 + 110 = 166

Total number of outcomes = 200

∴P(getting at least one head) = $\frac{166}{200}$ = 0.83

II. Short answer type questions

1. A bag contains 12 balls out of which x balls are white. If one ball is taken out from the bag. Find the probability of getting a white ball. If 6 more white balls are added to the bag and the probability now for getting a white ball is double the previous one, find the value of x. [CBSE 2012, HOTS]

Sol. Total Number of balls = 12

Number of white balls = x

∴ P (getting a white ball) = $\frac{x}{12}$ = $P(E_1)$

Now, 6 more white balls are added in that bag

∴ Total number of balls = 12 + 6 = 18

∴ P (getting a white ball) = $\frac{6+x}{18}$ = $P(E_2)$

According to the given condition,

$$P(E_2) = 2P(E_1)$$

$$\frac{6+x}{18} = 2 \times \frac{x}{12}$$

$$\Rightarrow \quad \frac{6+x}{18} = \frac{x}{6}$$

$$\Rightarrow \quad 6 + x = 3x$$

$$\Rightarrow \quad 2x = 6$$

$$\Rightarrow \quad x = 3$$

2. A die is rolled 300 times and following outcomes are recorded

Outcomes	1	2	3	4	5	6
Frequency	42	60	55	53	60	30

Find the probability of getting a number (i) more than 4(ii) less than 3

Sol. (i) Number of possible outcomes to get a number more than 4 = 60 + 30 = 90

Total number of time die rolled = 300

∴ P (getting a number more than 4)

$= \frac{90}{300} = \frac{3}{10} = 0.3$

.(ii) Number of possible outcomes to get a number less than 3 = 42 + 60 = 102

∴ P (getting a number less than 3)

$= \frac{102}{300} = \frac{51}{150} = 0.34$

3. A purse contains a number of Rs.1, Rs.2 and Rs.5 coins as given below.

Rs.1	Rs.2	Rs.5
10	14	14

If from the purse a coin is taken out at random, then find the probability that the coin

(i) is not a Rs.1 coin (ii) is a Rs.3 coin [CBSE 2013]

Sol. Total number of coins = 10 + 14 + 14 = 38

(i) Total number of Rs.1 coin = 10

∴ P (Rs.1 coin) = $\frac{10}{38}$

But P (not a Rs.1 coin) = 1 - P (Rs.1 coin)

$= 1 - \frac{10}{38} = \frac{28}{38} = \frac{14}{19}$

(ii) Since there is no outcome favourable to choose Rs.3 coin

∴ P(a Rs.3 coin) = $\frac{0}{38} = 0$

4. A box has 4 red balls and 12 black balls. Find the probability that the selected ball is (i) a red ball (ii) a black ball, chosen at random from the box. Also prove that sum of these two probabilities is 1.

Sol. Total number of balls in the box = 4 + 12 = 16

(i) Number of red balls = 4

$\therefore$ P (a red ball) = $\frac{4}{16} = \frac{1}{4}$

(ii) Number of black balls = 12

$\therefore$ P (a black ball) = $\frac{12}{16} = \frac{3}{4}$

Consider P (a red ball) + P (a black ball)

$= \frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1$ Hence proved

I. Long answer type questions

1. The daily cost of milk (in Rs) supplied to 25 houses in a locality are given below

Cost (in Rs.)	Number of houses
40 – 50	4
50 – 60	5
60- 70	3
70 – 80	5
80 – 90	2
90 - 100	6

If one house is chosen at random, find the probability that

(i) The milk bill of the house lies between Rs.60 and Rs.80

(ii) house is paying at most Rs.69, for the milk bill.

(iii) the milk bill of the house is below Rs.50 **[CBSE 2013]**

Sol. Total number of houses = 25

(i) Total number of houses paying the milk bill between Rs.60 and Rs.80 = 3 + 5 = 8

$\therefore$ P (milk bill between Rs.60 and Rs.80) = $\frac{8}{25}$

(ii) Total number of houses paying at most Rs.69 for milk = 4 + 5 + 3 = 12

∴ P (milk bill at most Rs.69) = $\frac{12}{25}$

(iii) Total number of houses paying the bill for milk below Rs.50 = 4

∴ P (milk bill below Rs.50) = $\frac{4}{25}$

2. A parent has collected data of number of schools based on the monthly fees, so that the can choose the school for admission of his child. The data is as follows.

Monthly fees of schools (in Rs)	Number of schools
250 – 500	14
500 – 750	16
750 – 1000	18
1000 – 1250	12
1250 – 1500	14
1500 – 1750	8
1750 - 2000	8

If a school is selected at random, find the probability that the school is having

(i) minimum fee

(ii) maximum fee

(iii) fee lessthan Rs.1000

(iv) fee Rs.1000 or more but less than Rs.1500 [CBSE 2015 AND 2016]

Sol. Total number of schools

= 14 + 16 + 18 + 12 + 14 + 8 + 8 = 90

(i) Number of schools having minimum fee

(in the range Rs.250 – Rs.500) = 14

∴ P(School having minimum fee) = $\frac{14}{90}$ = $\frac{7}{45}$

(ii) Number of schools having maximum fee

(in the range Rs.1750 – Rs.2000) = 8

∴ P(School having maximum fee) = $\frac{8}{90}$ = $\frac{4}{45}$

(iii) Number of schools having fee less than Rs.1000= 14 + 16 + 18 = 48

∴ P(School having fee less than Rs.1000)

$= \frac{48}{90} = \frac{24}{45} = \frac{8}{15}$

(iv) Number of schools having fees Rs.1000 or more but less than Rs.1500= 12 + 14 = 26

∴P(School having fee Rs.1000 or more but less than Rs.1500)

$= \frac{26}{90} = \frac{13}{45}$

3. The given table shows the marks obtained by 50 students out of 100 in history examination.

Marks obtained	Number of Students
0 – 25	9
25 – 50	8
50 - 75	23
75 – 100	10
Total	50

A students is chosen at random

(i) Find the probability that he has obtained 75 or more marks.

(ii) If 50% are passing marks,Find the probability of the students failing in history examination

(iii) Find the probability that the student has obtained less than 75 marks

[CBSE 2016]

Sol. Total number of students = 50

(i) Number of students who has obtained 75 or more marks = 10

∴P(a students has obtained 75 or more marks)

$= \frac{10}{50} = \frac{1}{5}$

(ii) Number of students who has obtained less than 50% marks = 9 + 8 = 17

∴P(a students filing in history examination)

$= \frac{17}{50}$

(iii) Number of students who has obtained less than 75 marks = 9 + 8 + 23 = 40

∴P(a students has obtained less than 75 marks)

$= \frac{40}{50} = \frac{4}{5}$

4. Two coins are tossed 729 times and the outcomes are recorded below

Outcomes	Frequency
No tails	189
One tails	297
Two tails	243

Find the probability of each event. Also, find the probability that at least one tails will come. **[CBSE 2015]**

Sol. Total number of outcomes = 729

The probability of each event is given by

$$P(\text{no tail}) = \frac{Frequency\ of\ no\ tail}{Total\ number\ of\ trials}$$

$$= \frac{189}{729} = \frac{7}{27}$$

$$P(\text{one tail}) = \frac{Frequency\ of\ one\ tail}{Total\ number\ of\ trials}$$

$$= \frac{297}{729} = \frac{11}{27}$$

$$P(\text{two tail}) = \frac{Frequency\ of\ \text{two}\ tail}{Total\ number\ of\ trials}$$

$$= \frac{243}{729} = \frac{1}{3}$$

Number of possible outcomes for getting at least one tail = 297 + 243 = 540

$\therefore$ P (getting at least one tail) $= \frac{540}{729} = \frac{20}{27}$