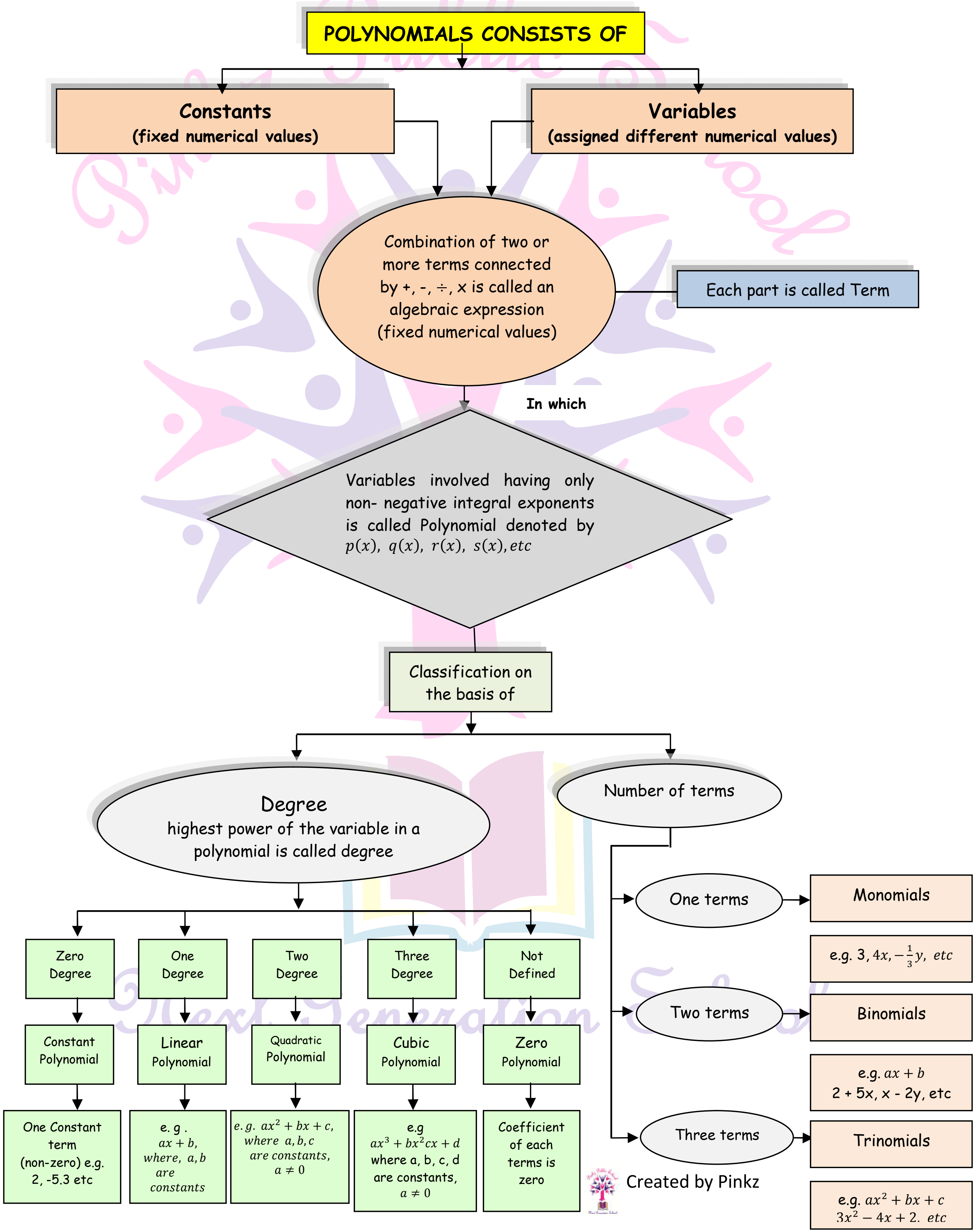
Grade IX

Lesson :2POLYNOMINALS

INTRODUCTION TO POLYNOMIALS AND PLOYNOMIALS IN TWO VARIABLES



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ZEROES OF A POLYNOMIAL

ZEROES (ROOTS) OF A POLYNOMIAL

A number x = a is called zero of the polynomial $p(x)$, *if* $p(a) = 0$

Can be obtained by

Hit and trial method

By putting x = a in a given polynomial and check
(i) If $p(a) = 0$, *then 'a' is a zero of the given polynomial*
(ii) If $p(a) \neq 0$, *then 'a' is not a zero of the given polynomial*

Example:

- Verify whether 3 and 0 are zeroes of the polynomial $2x^2 - 3x$

Sol. Let $p(x) = 2x^2 - 3x$
Then $p(3) = 2 \times 3^2 - 3 \times 3 = 18 - 9 = 9 \neq 0$
$p(0) = 0 - 0 = 0$
Hence, 0 is a zero of the polynomial $2x^2 - 3x$, but 3 is not

Solving the polynomial equation by taking $p(x) = 0$

Example: Let $p(x) = 2x + 3$
Its root is given by $p(x) = 0 \implies 2x + 3 = 0 \implies x = -\frac{3}{2}$
Thus, x = $-\frac{3}{2}$ *is a zero of* $p(x) = 2x + 3$

Its properties are

- Every linear polynomial has unique zero
- A non-zero constant polynomial has no zero
- A ploynomial can have more than one zero depending upon its nature.
- Every real number is a zero of the zero polynomial
- A zero of a polynomial need not be zero
- 0 may be the zero of a polynomial.

THEOREMS IN POLYNOMIALS

THEOREMS IN POLYNOMIALS

Remainder theorem

If p(x) be any polynomial of degree, n ≥ 1, and 'a' be any real number. When p(x) is divided by linear polynomial g(x) = $(x - a)$ the remainder is y(x) = $p(a)$ and focuses on finding the remainder even if p (x) is not divided by f(x)

Example : Find the remainder when the polynomial p(y) = y^4 - 2y^2 + 3y – 1 is divided by g(y) = y – 1

Sol. Here, the zero of y – 1 is 1. By remainder theorem, the remainder is :

$p(1) = 1^4 - 2 \times 1^2 + 3 \times 1 - 1$

= 1 – 2 + 3 -1

= 1

Factor theorem

If $p(x)$ is a polynomial of degree, n≥ 1 and 'a' is any real number, then

(i) $x - a$ *is a factor of* $p(x)$ *if* $p(a) = 0$ *and*

(ii) $p(a) = 0$ *if* ($x - a$ *is a factor of* $p(x)$)

It focuses on

- Checking operation in division of number
- Finding linear factor of $p(x)$

Example

$p(x) = x^3 - 4x^2 - 3x + 18$

Sol. For x-3 to be a factor of $p(x)$*, by factor theorem,* $p(3)$ *should be zero, where 3 is the zero of* $x - 3$

Now, by remainder theorem,

$p(3) = 3^3 - 4.3^2 - 3.3 + 18$

$= 27 - 36 - 9 + 18$

$= 45 - 45 = 0$

Hence, by Factor theorem, x-3 is a factor of the given polynomial $p(x)$

FACTORISATION OF POLYNOMIALS

MONOMIALS

- No factor

BINOMIALS

- Two Perfect squares
- Two perfect cubes

TRINOMIALS

Quadratic polynomials $ax^2 + bx + c$

1. By middle term spilliting

p + q = b, pq = ac

$(i)\ \ ax^2 + bx + c = x^2 + (p+q)x + pq$

$(ii)\ \ ax^2 + bx + c = (x+p) + (x+q)$

$(iii)\ ax^2 - bx + c = (x-p) + (x-q)$

$(iv)\ ax^2 \pm bx - c = (x+p) + (x-q)$

Note : in point (iv).

- If b is negative, then negative sign goes with bigger number.
- If b is positive, then negative sign goes with smaller number.

2. If quadratic, polynomial is a perfect square, then use

$x^2 + 2xy + y^2 = (x+y)^2$

$x^2 - 2xy + y^2 = (x-y)^2$

3. By using factor theorem.

CUBIC POLYNOMIALS

$p(×)= ax^3 + b^2 + cx + d$

- It has one real or distinct real root
- One pair of repeated roots and a district roots

Can be found by using the following steps:

- Write all factors of constant term
- By trial, find $p(q) = 0$
- (x - q) is a factor of $p(x)$
- Divide $p(x)\ by\ (x - q)$
- Quotient is a quadratic equation.
- Factorise quadratic equation.

$\therefore ax^3 + bx^2 + cx + d = (x-p)(x-q)(x-r)$

ALGEBRAIC IDENTITIES

Identity I	:	$(x+y)^2 = x^2 + 2xy + y^2$
Identity II	:	$(x-y)^2 = x^2 - 2xy + y^2$
Identity III	:	$x^2 - y^2 = (x+y)(x-y)$
Identity IV	:	$(x+a)(x+b) = x^2 + (a+b)x + ab$
Identity V	:	$(x+y+z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$
Identity VI	:	$(x+y)^3 = x^3 + y^3 + 3xy(x+y)$
Identity VII	:	$(x-y)^3 = x^3 - y^3 - 3xy(x-y)$
		$= x^3 - 3x^2y + 3xy^2 - y^3$
Identity VIII	:	$x^3 + y^3 + z^3 - 3xyz = (x+y+z)(x^2 + y^2 + z^2 - xy - yz - zx)$

Objective Type Questions

I. Multiple choice questions 2.1

1. The polynomial $2x - x^2$+5 is

a) an equation b) a trinomial c) a binomial d) a monomial

Sol: b) a trinomial

2. Degree of a zero polynomial is (NCERT)

a) 0 b) 1

c) any natural number d) not defined

Sol: d) because zero polynomial means coefficient of any variable, i.e. $0x^2$, $0x^3$, $0x^4$, $0x^5$, $0x^2$,........... is zero.

3. Write True or False and justify your answer. The degree of the sum of two polynomials each of degree 5 is always 5. (NCERT)

Sol: False, because x^5 + 1 and -x^5+ $3x$are two polynomials each of degree 5, but the degree of the sum of the two polynomials is 1.

4. Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer.

i) 3y^2 + 4y ii) 2$t^2 + 5$ iii) y + $\frac{1}{y}$iv) $\sqrt{x}$ + 4

v) -x^2 + 3x^2 + 5$x - 3$vi) 3$x^2 - 5x$ + 8 vii) $\sqrt[3]{t}$ + t^2 viii) 4 $\sqrt{t}$ + $t\sqrt{3}$

Sol: i), ii), v) and vi) are polynomials in one variable because these expressions contain one variable only. iii) iv), vii) and viii) are not polynomials because the exponent of the variable in these expressions are not a whole number.

5. Write the coefficients ofx^2 in each of the following:

i) a$x^2 +$ bx + c ii) 4$x^2 + 5x$ - 3 iii) 3-5x^2+ x^3 iv) $\sqrt{2}x^2$+4y + 5

The coefficients of x^2 in each of the given polynomials are

Sol: i) a ii) 4 iii) -5 iv) $\sqrt{2}$

6. Write the coefficients of x^3 in each of the following:

i) $5x^3 - 6x^2 + 7x - 9$ **ii) $27x^3 - Y^3$** **iii) $\frac{2}{5}x^3 - x$**

iv) $2x^3 + 5$ **v) $x^3 + x + 6$** **vi) $\sqrt{5}x^3 + 1$**

vii) $Y^3 - 16x^3$ **viii) $3x^4 - 4x^3 - 3x - 5$**

Sol: The coefficients of x^3 in the given expressions are :

i) 5 ii) 27 iii) $\frac{2}{5}$ iv) 2

v) 1 vii) $\sqrt{5}$vi) -16 vii) -4

7. Write the degree of each of the following polynomials:

i) $4x^5 - 3x^4 + 9$ **ii) $3 - 2y^2 - 5y^3 + 2y^8$**

iii) 3 **iv) 4y + 5**

Sol: The highest power of the variable in a polynomial is called the degree of the polynomial. Hence, degree of the given expression are

i) 5 ii) 8 iii) 0 ($\because$ 3 = $3x^0$) iv) 1

8. Write the coefficient of y in the expansion of $(5 - y)^2$

Sol: (5-y)= 5^2 - 2 x 5x y + y^2

[Using $(a-b)^2$ = a^2 - 2ab -b^2]

= 25 – 10 y + y^2

$\therefore$ Required coefficient of y = -10

II. Multiple choice questions 2.2

1. Given a polynomial p(t) = t^4 - t^3 + t^2+6, then p(-1) is

(a) 6 **(b) 9** **(c) 3** **(d)-1**

Sol: P(t) = t^4 - t^3 + t^2+ 6

$\Longrightarrow$ p (-1) = $(-1)^4$- $(-1)^3$ + $(-1)^2$ + 6

= 1 – (-1) + 1 + 6 = 1 + 1 + 7 = 9

∴ Correct option is (b)

2. Zero of the zero polynomial is

a) 0 **b) 1** **c) any real number** **d) not defined**

Sol: c) Consider g(x) = 0 (x – a) where 'a' is any real number.

Zero of g(x) is equal to g(x)= 0 $\Longrightarrow$ x - a = 0 $\Longrightarrow$ x – a

So, zero of the zero polynomial is any real number.

3. If p(x) = $x + 3$, then p (x) + p ($-x$) is equal to

a) 3 **b) 2x** **c) 0** **d) 6**

Sol: p (x) + p ($-x$) = $x + 3$ + (-x) $+ 3 = 6$

∴ Correct option is (d)

4. Find the value of polynomial y^2 – 5y + 6 at

i) y = 0 **ii) y = -1**

Sol: The value of polynomial p(y) = y^2 – 5y + 6

i) at y = 0 is given by

p(0) = 0 – 5 (0) + 6 = 6

ii) at y = -1 is given by

p(-1) = $(-1)^2$ – 5 (-1) + 6 = 1 + 5 + 6 = 12

5. **Find the value of each of the following polynomials at the indicated value of variable:**

i) p(y) = $5y^2$ - 3y + 7 at y = 1, -1

ii) q(x) = $3x^3$ - $4x$ + $\sqrt{8}$ at x = 2

Sol: i) p (y) = $5y^2$ - 3y + 7

∴ At y = 1, p(1) = 5 $(1)^2$ - 3(1) + 7 = 5 -3 + 7 = 9

and at y = -1 , p(-1) = 5 $(-1)^2$ - 3(-1) + 7 = 5 + 3 + 7 = 15

ii) q(x) = $3x^3$ - $4x$+ $\sqrt{8}$

∴ at$x$ = 2, q =(2) = $3(2)^3$ - 4 x 2 + $\sqrt{8}$= 24 – 8 + $\sqrt{8} = 16 + \sqrt{8}$

6. **Verify whether the following are zeroes of the polynomial, indicated against them.**

i) p (x) = x^3 - $3x^2$ + 4x – 12, $x = 3$

ii) p (y) = y^4 - $3y^2$ + 2y + 1 , $y = 1$

Sol: i) p (x) = x^3 - $3x^2$ + 4x – 12,

∴ p(3) = 3^3 - 3 $(3)^2$ + 4 (3) – 12 = 27 – 27 + 12 – 12 = 0

So, x = 3 is a zero of the polynomialp(x) = x^3 - $3x^2$ + 4x – 12.

ii) p (y) = y^4 - $3y^2$ + 2y + 1

∴ p(1) = 1^4 - 3 $(1)^2$ + 2 (1) + 1 = 1 – 3+ 2 + 1 = 1 $\neq$ 0

So, y = 1 is not a zero of the polynomialp(y) = y^4 - $3y^2$ + 2y + 1

III. Multiple choice questions 2.3 and 2.4

1. **If p(x) = (x - 1) $(x + 2)$, then we say,**

(a) $(x - 1)$**is a factor of p**(x) **(b)** $(x + 2)$ **is a factor of p**(x)

(c) p (x) **is divisible by both (**x **- 1) and** $(x + 2)$ **(d) All of these**

Sol: (d) All of these

2. Factors of $3x^2 - x - 4$ are

a) $(x-1)$and$(3x-4)$ b) $(x+1)$ and $(3x-4)$

c) $(x+1)$and$(3x+4)$ d) $(x-1)$ and $(3x+4)$

Sol: $3x^2 - x - 4 = 3x^2 + 4x + 3x - 4$

$= x(3x-4) + 1(3x-4) = (x+1)(3x-4)$

∴ correct option is (b)

IV. Multiple choice questions 2.5

1. Expansion of $(x-y)^3$ is

a) $x^3+y^3+3x^2y+3xy^2$ b) $x^3+y^3-3x^2y-3xy^2$

c) $x^3-y^3-3x^2y+3xy^2$ d) $x^3-y^3-3x^2y-3xy^2$

Sol: c

2. Which identity, do we use to factorise $x^2 - \frac{y^2}{100}$?

a) $(a+b)^2 = a^2 + b^2 + 2ab$ b) $(a-b)^2 = a^2 + b^2 - 2ab$

c) $(a-b)^3 = a^3 + 3a^2b + 3ab^2 - b^3$ d) $a^2 - b^2 = (a-b)(a+b)$

Sol: d

3. To find the value of $(28)^3 + (-15)^3 - (-13)^3$, we use the formula

a) $x^3 + y^3 + z^3 - 3xyz = (x+y+z)(x^2+y^2+z^2 - xy - yz - zx)$

b) we calculate all the cubes and add.

c) we use $x^3 + y^3 + z^3 = 3xyz$ with $x = 28$ $y = -15$ $z = 13$

d) we use $x^3 + y^3 + z^3 = 3xyz$ with $x = 28$ $y = -15$ $z = -13$

Sol: d

4. Zeros of the polynomial $p(x) = (x-2)^2 - (x+2)^2$ are

a) 2, -2 b) $2x$ c) 0, -2 d) 0

Sol: $p(x) = (x-2)^2 - (x+2)^2 = x^2 + 4 - 4x - (x^2 + 4 + 4x)$

$= x^2 + 4 - 4x - x^2 - 4 - 4x = - 8x$

Now, $p(x) = 0 \Rightarrow -8x = 0 \Rightarrow x = 0$

∴ Correct option is (d)

I Short Answer Type Questions

1. If -1 is a zero of the polynomial p(x) = $ax^3 - x^2 + x + 4$, then find the value of 'a'.

Sol: If -1 is a zero of the polynomial p(x) , then

P (-1) = 0

$\Rightarrow$ a $(-1)^3 - (-1)^2 + (-1) + 4 = 0$

$\Rightarrow$ - a - 1 – 1 + 4 = 0

$\Rightarrow$ - a + 2 = 0 $\Rightarrow$ a = 2

2. What is the value of polynomial $x^2 + 8x$ + k. If -1 is a zero of the polynomial?

Sol: p(x) = $x^2 + 8x$ + k

If - 1 is a zero of the polynomial = p(x), then

p(-1) = 0 $\Rightarrow (-1)^2 + 8(-1) + k = 0$

1 – 8 + k = 0 $\Rightarrow$ k = 7.

II Short Answer Type Questions

3.If $x = -\frac{1}{3}$ is a zero of a polynomial. p(x) = $27x^3 - ax^2 - x + 3$, then find the value of 'a'.

Sol: Given, p(x) = $27x^3 - ax^2 - x + 3$

If $x = -\frac{1}{3}$ is a zero of p(x), then,

$p(-\frac{1}{3}) = 0 \Rightarrow 27\left(-\frac{1}{3}\right)^3 - a\left(-\frac{1}{3}\right)^2 - (-\frac{1}{3}) + 3 = 0$

$\Rightarrow$ - 27 x $\frac{1}{27}$ - $\frac{a}{3}$ + $\frac{1}{3}$ + 3 = 0

$\Rightarrow \frac{a}{3}$ = -1 + $\frac{10}{3}$ = $\frac{7}{3}$

$\Rightarrow$ a = 7

4. If f(x) = 5x^2- 4x + 5, find f(1) + f (-1) + f (O)

Sol: If f(x) = 5x^2 - 4x + 5

f(1) = 5$(1)^2$ -(4)(1) + 5 = 5 – 4 + 5 = 6

f(-1) = 5$(-1)^2$ -(4)(-1) + 5 = 5 + 4 + 5 = 14

f(0) = 5 $(0)^2$ - 4.0 + 5 = 0 – 0 + 5 = 5

∴f(1) + f (-1) + f(0) = 6 + 14 + 5 = 25

III. Short answer Type questions

1. Find the remainder when 4x^2 - 3x^2 + 4x - 2 is divided by

i) x -1 **ii) x -2**

Sol: i) Let p(x) = 4x^2 - 3x^2 + 4x – 2 and g(x) = x -1

Zero of g(x) is 1

By the remainder theorem, the remainder is, p(1)

= 4 x $(1)^3$ - 3 x 1^2 + 4 x 1 – 2

= 4 – 3 + 4 – 2 = 8 – 5 = 3

Hence, remainder is 3.

ii) Let p(x) = 4x^2 - 3x^2 + 4x – 2 and g(x) = x – 2

Here, zero of g(x) = 2 [g(x) = 0 $\Rightarrow$x = 2]

Using remainder theorem, when p(x) is divided by g(x) , we get the remainder p(2)

p(2) = 4 $(2)^3$ - 3 $(2)^2$ + 4(2) - 2 = 32 – 12 + 8 – 2 = 40 – 14 = 26.

Hence, remainder is 26

2. **By remainder theorem, find the remainder when p(y) is divided by g(y)**

i) p(y) = $4y^3$ - $12y^2$ + 5y – 4 and g(y) = 2y – 1

ii) p(y) = y^3 - $4y^2$ - 2y + 6 and g(y) = 1 - $\frac{3}{4}$ y.

Sol: i) Here, zero of g(y) is $\frac{1}{2}$ … [2y – 1 = 0 ⇒ y = $\frac{1}{2}$]

Using remainder theorem, when p(y) is divided by g(y) , we get the remainder p$\left(\frac{1}{2}\right)$

$\therefore$ p $\left(\frac{1}{2}\right)$ = 4$\left(\frac{1}{2}\right)^3$ - 12 $\left(\frac{1}{2}\right)^3$ + 5 $\left(\frac{1}{2}\right)$ - 4

$= \frac{4}{8} - \frac{12}{4} + \frac{5}{2} - 4$

$= \frac{4\ -\ 24\ +\ 20\ -\ 32}{8} = -\ \frac{32}{8} = -4$

Hence, required remainder is (-4).

ii) p(y) = y^3 - $4y^2$ - 2y +6 and g(y) = 1 - $\frac{3}{4}$ y.

Here, zero of g(y) is $\frac{4}{3}$ [1- $\frac{3}{4}y$ = 0 ⇒ y = $\frac{4}{3}$]

Using remainder theorem, when p(y) is divided by g(y) , the remainder, we get is p $\left(\frac{4}{3}\right)$

$\therefore$ p $\left(\frac{4}{3}\right)$ = $\left(\frac{4}{3}\right)^3$ - 4$\left(\frac{4}{3}\right)^2$ - 2 $\left(\frac{4}{3}\right)$+ 6

$= \frac{64}{27} - \frac{64}{9} - \frac{8}{3} + 6$

$= \frac{64\ -\ 192\ -\ 72\ -162}{27} = \frac{226-264}{27} = -\ \frac{38}{27}$

Hence, required remainder is $\left(-\frac{38}{27}\right)$.

3. **Check whether p(x) or not**

i) p(x) = x^3 + $27x^2$ - $8x$ + 18, g(x) = $(x-1)$

ii) p(x) =$2\sqrt{2}x^3$ – 5 $\sqrt{2}x^2$ + $7\sqrt{2}$, g(x) = $(x+1)$

Sol: g(x) = $(x-1)$ ∴ zero of g(x) is 1

Now, p(x) = x^3 + $27x^2$ - $8x$ + 18,

p(1) = 1^3 - 27$(1)^2$ - 8(1) + 18,

= 1 + 27 – 8 + 18

= 38 ≠ 0

Since p(1) $\neq$ 0, so p(x) is not a multiple of g(x)

ii) g(x) = $x + 1$ $\therefore$ zero of g(x) is -1

Now, p(x) = $2\sqrt{2}x^3$ - 5 $\sqrt{2}x^2$ + $7\sqrt{2}$

p(-1) = $2\sqrt{2}(-1)^3$- 5 $\sqrt{2}(-1)^2$ + $7\sqrt{2}$

= $-2\sqrt{2}$ - 5 $\sqrt{2}$ + $7\sqrt{2}$ = 0

Since p(1) = 0 then p(x) is a multiple of g(x)

4. Examine whether x-1 is a factor of the following polynomials:

i) $2x^3$ - $5x^2$ + x + 2 **ii) $4x^3$+ $5x^2$- $3x$+ 6**

Sol: If x – a is a factor of p(x), then by factor theorem, p(a) = 0.

Here for all expressions x = 1.

i) Let p(x) $= 2x^3 - 5x^2 + x + 2$

p(1) = 2 $(1)^3$ - 5$(1)^2$ + 1+ 2

=2 – 5 + 1 + 2 = 5 - 5 = 0

Hence, $x - 1$ is a factor of p(x) = $2x^3$ - $5x^2$ + x + 2

ii) Let p(x) = $4x^3$+$5x^2$- $3x$+ 6

p(1) = 4 $(1)^3$+5$(1)^2$-3(1)+6

= 4 + 5 – 3 + 6 = 15 – 3 = 12

$\Rightarrow$p(1) = 12$\neq$ 0 Hence, x – 1 is not a of p(x) = $4x^3$+$5x^2$- $3x$+ 6

5. Find the value of k, if $x +$k is the factor of the polynomials:

Sol: **i) x^3+ kx^2 -2 x + k + 5** **ii) x^4 – k^2x^2 + 3 x - 6k**

Using factor theorem, if x + k is a factor of p(x) then p(-k) =0. Here for all expressions $x = -k$.

i) Let p(x) = x^3+kx^2 -2x+k + 5

$\therefore$ p(-k) = $(-k)^3$ + k $(-k)^2$ -2 (-k) + k + 5 = 0

$\Rightarrow$ - k^3 + k^3 + 3k + 5 = 0

$\Rightarrow$ k = - $\frac{5}{3}$

ii) Let p(x) = x^4–k^2x^2 + 3x- 6k

$p(-k) = (-k)^4 - k^2.(-k)^2 + 3(-k) - 6k = 0$

$\Rightarrow k^4 - k^4 - 3k - 6k = 0$

$\Rightarrow -9k = 0 \Rightarrow k = 0$

IV. Short answer Type questions

6. Let R_1 and R_2 are the remainders when polynomial f(x)= $4x^3 + 3x^2 + 12ax - 5$ and g(x) = $2x^3 + ax^2 - 6x - 2$ are divided by (x-1) and (x-2) respectively. If $3R_1 + R_2 - 28 = 0$, Find the value of a. [CBSE 2015, HOTS]

Sol: Given, $f(x) = 4x^3 + 3x^2 + 12ax - 5$

$g(x) = 2x^3 + ax^2 - 6x - 2$

Now, R_1 = remainder when $f(x)$ is divided by x – 1

$\Rightarrow R_1 = f(1)$

$\Rightarrow R_1 = 4(1)^3 + 3(1)^2 + 12a(+1) - 5$

= 4 + 3 + 12a - 5 =12a + 2

And R_2 = remainder when $g(x)$ is divided by x – 2

$\Rightarrow R_2 = g(2)$

$= 2(2)^3 + a(2)^2 + 6(2) - 2$

= 16 + 4a – 12 – 2 = 4a + 2

Substituting the value of R_1 and R_2 are in $3R_1 + R_2$ = 28 = 0, we get

3(12a + 2) + 4a + 2 – 28 = 0

36a + 6 + 4a – 26 = 0

40a = 20

a = $\frac{20}{40} = \frac{1}{2}$

7. Divide $3y^4 - 8y^3 - y^2 - 5y - 5$ by $y - 3$ and find the quotient and the remainder. [CBSE2014]

Sol:

$$
\begin{array}{r|l}
 & 3y^3 + y^2 + 2y + 1 \\
\hline
y - 3 & 3y^4 - 8y^3 - y^2 - 5y - 5 \\
 & 3y^4 - 9y^3 \\
 & - \quad + \\
\hline
 & y^3 - y^2 \\
 & y^3 - 3y^2 \\
 & - \quad + \\
\hline
 & 2y^2 - 5y \\
 & 2y^2 - 6y \\
 & - \quad + \\
\hline
 & y - 5 \\
 & y - 3 \\
 & - \quad + \\
\hline
 & -2 \\
\hline
\end{array}
$$

$\therefore$ $Quotient = 3y^3 + y^2 + 2y + 1$

Remainder = -2

V. Short answer Type questions

1. Find the following products by using suitable identities.

(i) $(2x + 5)(2x - 5)$ (ii) $(x^2 + 4)(x^2 - 4)$

(iii) $(4 + 5x)(4 + 5x)$ (iv) $(3x + 4y)(3x - 8y)$

Sol : (i) $(2x + 5)(2x - 5) = (2x)^2 - 5^2$

$= 4x^2 - 25$ [Using (a+b)(a-b)= $a^2 - b^2$]

(ii) Using identity (a+b)(a-b)=$a^2 - b^2$

We have $(x^2 + 4)(x^2 - 4) = (x^2)^2 - (4)^2 = x^4 - 16$

(iii) Using identity $(a + b)^2 = a^2 + 2ab + b^2$

We have $(4+5x)(4+5x)$ = $(4+5x)^2$

= $4^2 + 2(4)(5x) + (5x)^2$

= 16 + 40x +$25x^2 = 25x^2 + 40x + 16$

(iv) Using identity $(x+a)(x+b)$= $x^2 + (a+b)x + ab$

We have, $(3x+4y)(3x-8y)$

= $(3x)^2 + [(4y) + (-8y)](3x) + (4y)(-8y)$

= $9x^2 + (-4y)(3x) - 32y^2 = 9x^2 - 12xy - 32y^2$

2. Evaluate the following products by using suitable identities.

(i) 104×105 (ii) 102×98 (iii) $116 \times 116 - 106 \times 106$

Sol : (i) $104 \times 105 = (100+4) \times (100+5)$

= $100^2 + (4+5)$x 100 + 4 x 5

[Using $(x+a)(x+b) = [(x^2) + (a+b)x + ab]$

= 10000 + 900 + 20 = 10920

(ii) 102×98 = (100+2) (100-2)

= $(100)^2 - (2)^2$

[Using $(\mathrm{x}+\mathrm{y})(\mathrm{x}-\mathrm{y}) = \mathrm{x}^2 - \mathrm{y}^2$]

= 10000 - 4=9996

(iii) $116 \times 116 - 106 \times 106$ = $(116)^2 - (106)^2$

= (116+106) (116-106)

= $[Using\ a^2 - b^2 = (a+b)(a-b)]$

= 222 X 10 = 2220

3. Factorise the following using suitable identities:

(i) $x^2 - y^2 + 2x + 1$

(ii) $9a^2 - 4b^2 - 6a + 1$

(iii) $a^4 - 16b^4$

Sol : (i) $x^2 - y^2 + 2x + 1 = (x^2 + 2x + 1) - y^2$

= $(x + 1)^2 - y^2$ [Using $a^2 + 2ab + b^2 = (a + b)^2$]

= $(x + 1 + y)\,(x + 1 - y)$ [$(a^2 - b^2) = (a + b)(a - b)$]

= $(x + y + 1)\,(x - y + 1)$

(ii) $9a^2 - 4b^2 - 6a + 1 = (9a^2 - 6a + 1) - 4b^2$

= $[(3a)^2 - 2(3a)(1) + (1)^2] - (2b)^2]$

[Using $x^2 - 2xy + y^2 = (x - y)^2$]

= $(3a - 1)^2 - (2b)^2$

= $(3a - 1 + 2b)\,(3a - 1 - 2b)$

$[Using\ x^2 - y^2 = (x + y)(x - y)]$

= $(3a + 2b - 1)\,(3a - 2b - 1)$

(iii) $a^4 - 16b^4 = (a^2)^2 - (4b^2)^2$

= $(a^2 + 4b^2)(a^2 - 4b^2)$

$[Using\ x^2 - y^2 = (x + y)(x - y)]$

= $(a^2 + 4b^2)[(a)^2 - (2b)^2]$

= $(a^2 + 4b^2)(a + 2b)(a - 2b)$

4. Factorise the following

(i) $3\sqrt[2]{3}a^3 + 8b^3 - 27c^3 + 18\sqrt[2]{3}abc$

(ii) $a^3 - 8b^3 + 1 + 6ab$

(iii) $(a+b)^3 - (a-b)^3$

Sol : (i) $3\sqrt[2]{3}a^3 + 8b^3 - 27c^3 + 18\sqrt[2]{3}abc$

= $(\sqrt{3}a)^3 + (2b)^3 + (-3c)^3 - 3(\sqrt{3}a)(2b)(-3c)$

= $(\sqrt{3}a + 2b - 3c)\, x \left[(\sqrt{3}a)^2 + (2b)^2 + (-3c)^2 - (\sqrt{3}a)(2b) - (2b)(-3c) - (-3c)(\sqrt{3}a)\right]$

[By Using, $x^3 + y^3 + z^3 - 3xyz = (x+y+z)(x^2 + y^2 + z^2 - xy - yz - zx)$]

= $(\sqrt{3}a + 2b - 3c)(3a^2 + 4b^2 + 9c^2 - 2\sqrt{3}\, ab + 6bc + 3\sqrt{3}\, ca)$

(ii) $a^3 - 8b^3 + 1 + 6ab$

= $a^3 + (-2b)^3 + (1)^3 - 3(a)(-2b)(1)$

= (a - 2b + 1) $[(a)^2 + (-2b)^2 + (1)^2\, a(-2b) - (-2b)(1) - (a)(1)]$

= (a - 2b + 1) $(a^2 + 4b^2 + 1 + 2ab + 2b - a)$

(iii) Using identity $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

We have, $x = a + b \text{ and } y = a - b$

So, $(a+b)^3$- $(a-b)^3 = [(a+b) - (a-b)]$

$[(a+b)^2 + (a+b)(a-b) + (a-b)^2]$

= $(a + b - a + b)(a^2 + b^2 + 2ab + a^2 - b^2 + a^2 + b^2 - 2ab)$

= $2b\,(3a^2 + b^2)$

VI. Short answer Type questions

5. Prove that $(x+y)^3 + (y+z)^3 + (z+x)^3 - 3(x+y)(y+z)(z+x) = 2(x^3+y^3+z^3-3xyz)$

Sol : We known $a^2+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$ ..... (i)

Let a = x + y, b = y + z and c = z + x

So, a + b +c = (x + y) + (y + z) + (z + x)

= 2x + 2y + 2z = 2 (x + y + z)

and $a^2+b^2+c^2-ab-bc-ca$

= $(x+y)^2 + (y+z)^2 + (z+x)^2 - (x+y)(y+z) - (y+z)(z+x) - (z+x)(x+y)$

= $x^2+y^2+2xy+y^2+z^2+2yz+z^2+x^2+2zx-xy-xz-y^2-yz-yz-xy-z^2-zx-zx-zy-x^2-xy$

= $x^2+y^2+z^2+2xy+2yz+2zx-3xy-3yz-3zx$

= $x^2+y^2+z^2-xy-yz-zx$

Substituting the value in (i) we get

$(x+y)^3 + (y+z)^3 + (z+x)^3 - 3(x+y)(y+z)(z-x)$

= $2(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$

= $2(x^3+y^3+z^3-3xyz)$Hence proved.

6. If 2x+3y = 12 and xy = 6, find the value of $8x^3 + 27y^3$. **[CBSE 2016]**

Sol : We know that $(x+y)^3 = x^3+y^3+3xy(x+y)$

$\Rightarrow \quad x^3+y^3 = (x+y)^3 - 3xy(x+y)$

Now, $8x^3 + 27y^3 = (2x)^3 + (3y)^3$

= $(2x+3y)^3 - 3(2x)(3y)(2x+3y)$

= $12^3 - 18 \times 6 \times 12$ $[Given\ 2x+3y = 12\ and\ xy = 6]$

= 1728 -1296 = 432

Hence, $8x^3 + 27y^3 = 432$

7. If $z^2 + \frac{1}{z^2} = 34$, find the value of $z^3 + \frac{1}{z^3}$ using only the positive value of $z + \frac{1}{z}$ [CBSE2016]

Sol : Using identity

$$(x+y)^2 = x^2 + 2xy + y^2$$

We have $\left(z + \frac{1}{z}\right)^2 = z^2 + 2z.\frac{1}{z} + \frac{1}{z^2}$

$\Rightarrow \quad = \left(z^2 + \frac{1}{z^2}\right) + 2 = 34 + 2 = 36$

$\Rightarrow \quad = z + \frac{1}{z} = \pm\sqrt{36} = \pm 6$

According to the given condition, we should use only positive value of $z + \frac{1}{z}$

$\therefore \quad z + \frac{1}{z} = 6$

Now, again using the identity

$$x^3 + y^3 = (x+y)^3 - 3xy(x+y)$$

We have $z^3 + \frac{1}{z^3} = \left(z + \frac{1}{z}\right)^3 - 3z.\frac{1}{z}\left(z + \frac{1}{z}\right)$

$= 6^3 - 3 \times 6 = 216 - 18 = 198$

I. Long answer Type questions

1. Find the value of a and b so that x +1 and x – 1 are factors of $x^4 + ax^3 + 2x^2 - 3x + b$.

Sol : Let $f(x) = x^4 + ax^3 + 2x^2 - 3x + b$ be the given polynomial and $g(x) = x + 1, h(x) = x - 1$ $g(x)$ is a factor of $f(x)$, then by factor theorem.

$f(-1) = 0$

$\Rightarrow \quad (-1)^4 + a(-1)^3 + 2(-1)^2 - 3(-1) + b = 0$

$\Rightarrow \quad 1 - a + 2 + 3 + b = 0$

$\Rightarrow \quad -a + b = -6$....(i)

If $h(x)$ be a factor of $f(x)$, then again by factor theorem.

$f(1) = 0$

$\Rightarrow \quad (1)^4 + a(1)^3 + 2(1)^2 - 3(1) + b = 0$

$\Rightarrow \quad 1 + a + 2 + 3 + b = 0$

$\Rightarrow \quad a + b = 0$....(ii)

Adding (i) and (ii), we get

2b = -6 or b=-3

From (ii), we have a – 3 = 0, ⇒ a = 3

Hence, required value of a and b are 3 and -3 respectively.

2. State factor theorem. Using this theorem, factorise $x^3 - 3x^2 - x + 3$ [CBSE 2011]

Sol : Factor theorem : if $p(x)$ is a polynomial of degree n ≥ 1 and a is any real number, then

(i) x = a is a factor of $p(x)$ if $p(a)$= 0 and

(ii) $p(a)$ = 0 if $(x - a)$ is a factor of $p(x)$

Let $\quad p(x) = x^3 - 3x^2 - x + 3 \quad$ ..... (i)

Factors of 3 are ± 1 *and* $\pm\ 3$.

By trial method. $\quad p(1) = 1^3 - 3(1)^2 - 1 + 3$

= 1 - 3 - 1 + 3 = 0

Therefore, by factor theorem, $(x - 1)$ is a factor of $p(x)$

Similarly, $\quad p(-1) = (-1)^3 - 3(-1)^2 - (-1) + 3$

= - 1 – 3 + 1 + 3 = 0

So, (x+1) is also a factor of $p(x)$ and $p(3) = (3)^3 - 3(3)^2 - 3 + 3 = 27 - 27 - 3 + 3 = 0$

So, (x-3) is also a factor of $p(x)$

Since $p(x)$ is a polynomial of degree, 3, so it cannot have more than three linear factors.

Let $\quad p(x) = k\,(x - 1)(x + 1)(x - 3)$ ..... (ii)

$\Rightarrow \quad x^3 - 3x^2 - x + 3 = k\,(x - 1)(x + 1)(x - 3)$

Putting x = 0 both sides, we get

3 = $k\,(-1)(1)(-3) = 3k$

$\Rightarrow \qquad k = 1$

Putting k = 1 in equation (ii), we get

$f(x) = (x-1)(x+1)(x-3)$

3. Divide polynomial $p(x) = 3x^4 + 4x^3 + 4x^2 - 8x + 1$ by $q(x) = 3x + 1$, Also, find what should be added to $p(x)$ so that it is completely divisible by $q(x)$

$$
\begin{array}{r|l}
 & x^3 + x^2 + x - 3 \\
\hline
3x + 1 & 3x^4 + 4x^3 + 4x^2 - 8x + 1 \\
 & 3x^4 + x^3 \\
 & - \quad - \\
\hline
 & 3x^2 + 4x^2 \\
 & - \quad - \\
 & 3x^3 + x^2 \\
 & - \quad - \\
\hline
 & 3x^2 - 8x \\
 & 3x^3 + x \\
 & - \quad - \\
\hline
 & -9x + 1 \\
 & -9x - 3 \\
 & + \quad + \\
\hline
 & 4 \\
\hline
\end{array}
$$

$\therefore \qquad Quotient = x^3 + x^2 + x - 3 = g(x)$

Remainder = 4 = $r(x)$

Now, we know that

$p(x) = (x-a)g(x) + r(x)$

$\Rightarrow$ For $r(x)$ to be zero, then we must added -4 to $p(x)$ so that $p(x)$ must be exactly divisible by $g(x)$

II. Long answer Type questions

1. **Factorise the following** **[CBSE 2016]**

(i) $x^2 - \frac{y^2}{9}$ (ii) $2x^2 - 7x - 15$

Sol : (i) $x^2 - \frac{y^2}{9} = x^2 - \left(\frac{y}{3}\right)^2 = \left(x - \frac{y}{3}\right)\left(x + \frac{y}{3}\right)$

$[x^2 - y^2 = (x + y)(x - y)]$

(ii) $2x^2 - 7x - 15 = 2x^2 - 10x + 3x - 15$

$= 2x(x - 5) + 3(x - 5)$

$= (x - 5)(2x + 3)$

2. **If ab + bc + ca = 0, find the value of**

$\frac{1}{a^2 - bc} + \frac{1}{b^2 - ca} + \frac{1}{c^2 - ab}$ **[CBSE 2016, HOTS]**

Sol : Given : ab + bc + ca = 0

$\Rightarrow -ab = bc + ca; -bc = ca + ab \text{ and } -ca = ab + bc$

Therefore, $\frac{1}{a^2 - bc} + \frac{1}{b^2 - ca} + \frac{1}{c^2 - ab}$

$= \frac{1}{a^2 + ab + ca} + \frac{1}{b^2 + ab + bc} + \frac{1}{c^2 + bc + ca}$

$= \frac{1}{a\,(a + b + c)} + \frac{1}{b\,(b + a + c)} + \frac{1}{c\,(c + b + a)}$

$= \frac{1}{(a + b + c)}\left[\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right]$

$= \frac{1}{(a + b + c)}\left(\frac{ab + bc + ca}{abc}\right)$

$= \frac{0}{abc\,(a + b + c)} = 0$ as ab + bc + ca = 0

3. **It is given that 3a + 2b = 5c, then find the value of $27a^3 + 8b^3 - 125c^3$ *if* $abc = 0$ [CBSE 2016]**

Sol : Given: 3a + 2b = 5c

Taking cube both sides, we have

$(3a + 2b)^3 = (5c)^3$

$\Rightarrow (3a)^3 + (2b)^3 + 3(3a)(2b)(3a + 2b) = 125c^3$

[Using: $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$]

$\Rightarrow 27a^3 + 8b^3 + 18ab\,(3a + 2b) = 125c^3$

$\Rightarrow 27a^3 + 8b^3 + 18ab(5c) = 125c^3$

$[\because 3a + 2b = 5c]$

$\Rightarrow 27a^3 + 8b^3 + 90abc = 125c^3$

$\Rightarrow 27a^3 + 8b^3 + 90 \times 0 = 125c^3$

$\Rightarrow 27a^3 + 8b^3 - 125c^3 = 0$ $[\because abc = 0]$

4. If a + b + c = 0, then prove that $\frac{(b+c)^2}{3bc} + \frac{(c+a)^2}{3ac} + \frac{(a+b)^2}{3ab} = 1$

Sol : L.H.S. = $\frac{(b+c)^2}{3bc} + \frac{(c+a)^2}{3ac} + \frac{(a+b)^2}{3ab}$

$\frac{b^2 + c^2 + 2bc}{3bc} + \frac{c^2 + a^2 + 2ac}{3ac} + \frac{a^2 + b^2 + 2bc}{3ab}$

[Using: $(x + y)^2 = x^2 + y^2 + 2xy$]

$= \frac{1}{3abc}[ab^2 + ac^2 + 2abc + bc^2 + ba^2 + 2abc + a^2c + b^2c + 2abc]$

$= \frac{1}{3abc}[ab^2 + ac^2 + bc^2 + ba^2 + a^2c + b^2c + 6abc]$

$= \frac{1}{3abc}[ab\,(b + a) + ac\,(c + a) + bc(c + b) + 6abc]$

$= \frac{1}{3abc}[ab\,(-c) + ac\,(-b) + bc\,(-a) + 6abc]$ $[\because a + b + c = 0]$

$= \frac{1}{3abc}[-abc - abc - abc + 6abc]$

$= \frac{3abc}{3abc} = 1 = R.H.S.$ Hence, proved.