$\qquad$

## Chapter :13. Direct and Inverse Proportions

## Objective Type Questions

## 1 Marks.

## I. Multiple choice questions

1. If the distance travelled by a rickshaw in one hour is 10 km , then the distance travelled with the same rickshaw with the same speed in one minute is
[NCERT Exemplar]
a. $\frac{250}{9} m$
b. $\frac{500}{9} m$
c. 1000 m
d. $\frac{500}{3} m$

2- A train is running at a speed of $75 \mathrm{~km} / \mathrm{hr}$. What distance will it cover in 20 minutes?
[NCERT Exemplar]
(a) 15 km
(b) 20 km
(c) 23 km
(d) 23 km
3. A machine manufactures 840 bottles in hours. Find the number of bottles it can manufacture in five Hours.
[NCERT Exemplar]
(a) 600
(b) 650
(c) 700
(d) 750
4. A truck needs 54 litres of diesel for corning. distance at 247 km . The diesel required by Ile truck to cover a distance of 550 km is
[NCERT Exemplar]
(a) 100 litre
(b) 50 litres
(c) 25.16 litres
(d) 25 litres
5. If 15 workers can build a wall in 48 hours. how man, workers will be required to do the um work in 30 hours?
[NCERT Exemplar]
(a) 15
(b) 14
(c) 24
(d) 30
6. By travelling at a speed of 48 kilometres pet host a car finish a certain journey in 10 hour. To cover the same distance in at hours the speed of the car should be
[NCERT Exemplar]
(a) $60 \mathrm{~km} / \mathrm{h}$
(b) $80 \mathrm{~km} / \mathrm{h}$
(c) $30 \mathrm{~km} / \mathrm{h}$
(d) $40 \mathrm{~km} / \mathrm{h}$
7.6 pipes are required to fill a tank in 1 hr 20 minutes. How long will it take if only 5 pipes of the same type are used?
[NCERT Exemplar]
(a) 2 hr 36 minutes
(b) 1 hr 36 minutes
(c) 2 hours
(d) 1 hr 30 minutes
8. Meenakshee cycles to her school at an average speed of $12 \mathrm{~km} / \mathrm{h}$ and takes 20 minutes to reach her school. If she wants to reach her school in 12 minutes, her average speed should be
[NCERT Exemplar]
(a) $\frac{20}{3} \mathrm{~km} / \mathrm{h}$
(b) $16 \mathrm{~km} / \mathrm{h}$
(c) $20 \mathrm{~km} / \mathrm{h}$
(d) $15 \mathrm{krn} / \mathrm{h}$
9. 100 persons had food provision for 24 days. If 20 persons left the place, the provision will last for :
[NCERT Exemplar]
(a) 30 days
(b) $\frac{96}{5}$ days
(c) 120 days
(d) 40 days
10. If $\frac{x}{y}=\frac{x}{y}$ and $\mathrm{x}_{2}=7.2, \mathrm{y}_{1}=8.8, \mathrm{y}_{2}=3.6$, then $\mathrm{x}_{1}$ is equal to :
[NCERT Exemplar]
(a) 17.6
(b) 14.4
(c) 9.9
(d) 4.4
11. A machine produces 1800 tools in 6 hours. The number of tools produced by it in 9 hours is :
[NCERT Exemplar]
(a) 2700
(b) 5400
(c) 3600
(d) 900
12. If two quantities $x$ and $y$ vary directly with each other, then :
[NCERT Exemplar]
(a) $\frac{x}{y}$ remains constant
(b) $\mathrm{x}-\mathrm{y}$ remains constant
(c) $x+y$ remains constant
(d) $x \times y$ remains constant
13. The rate 01 working power of two men are irk the ratio 35 . The number of days taken by them to finish a work will be in the ratio :
[NCERT Exemplar]
(a) $3: 5$
(b) $5: 3$
(c) $3: 8$
(d) $8: 3$
14. If two quantities $p$ and $q$ vary inversely with each other, then
[NCERT Exemplar]
(a) $\frac{p}{q}$ remains constant
(b) $\mathrm{p}+\mathrm{q}$ remains constant
(c) $p \times q$ remains constant
(d) p - q remains constant
15. A Positive number and its reciprocal are said to be
[NCERT Exemplar]
(a) in inverse proportion
(b) in direct proportion
(c) neither in direct proportion nor in inverse proportion
(d) same number
16. The number of teeth and the age of a person vary [NCERT Exemplar]
(a) Directly with each other
(b) Inversely with each other
(c) Neither directly nor inversely with each other.
(d) Sometimes directly and sometimes inversely with each other
17. Both $u$ and $v$ vary directly with each other. When $u$ is $10, v$ is 15 , which of the following is not a possible pair of corresponding values of $u$ and $v$ ?
[NCERT Exemplar]
(a) 2 and 3
(b) 8 and 12
(c) 15 and 20
(d) 25 and 37.5
18. Both $x$ and $y$ vary inversely with each other. When $x 10, y$ is 6 , which of the following is not a possible pair of corresponding values of $x$ and $y$ ?
[NCERT Exemplar]
(a) 12 and 3
(b) 15 and 4
(c) 25 and 2.4
(d) 45 and 1.3
19. Which of the following vary inversely with each other?
[NCERT Exemplar]
(a) Speed and distance covered
(b) Distance covered and tam fare
(c) Distance travelled and time taken
(d) Speed and time taken
20. Both $x$ and $y$ are in direct proportion, then $\frac{1}{x}$ and $\frac{1}{7}$ are
(a) In indirect proportion
(b) In inverse proportion
(c) Neither in direct nor in inverse proportion.
(d) Sometimes in direct and sometimes in inverse proportion.

| 1.d | 2.d | 3.c | 4.a | 5.c | 6.a | 7.b | 8.c | 9.a | 10.a |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 11.a | 12.a | 13.b | 14.c | 15.a | 16.d | 17.c | 18.d | 19.d | 20.b |

## II. Multiple choice questions

1. The number of teeth and the age of a person vary
(a) directly with each other.
(b) inversely with each other.
(c) neither directly nor
(d) sometimes directly and sometimes inversely with each other
2. Both $x$ andy vary inversely with each other. Whenx is $10, y$ is 6 , which of the following is not a possible pair of corresponding values of $x$ and $y$ ?
[NCERT Exemplar]
(a) 12 and 5
(b) 15 and 4
(c) 25 and 2.4
(d) 45 and 1.3
3. Meenakshee cycles to her school at an average speed of $12 \mathrm{~km} / \mathrm{h}$ and takes 20 minutes to reach her school. If she wants to reach her school in 12 minutes, heraverage speed should be [NCERT Exemplar]
(a) $\frac{20}{3} \mathrm{~km} / \mathrm{h}$
(b) $16 \mathrm{~km} / \mathrm{h}$
(c) $20 \mathrm{~km} / \mathrm{h}$
(d) $15 \mathrm{~km} / \mathrm{h}$
4. If two quantities x and y vary directly with each other, then
[NCERT Exemplar]
(a) $\frac{x}{y}$ remains constant.
(b) $x-y$ remains constant
(c) $x+y$ remains constant.
(d) $x \times y$ remains constant.
5. A truck needs 54 litres of diesel for covering a distance of 297 km . The diesel required by the truck to cover a distance of 550 km is ,
[NCERT Exemplar]
(a) 100 litres
(b) 50 litres
(c) 25.16 litres
(d) 25 litres
6. If two quantities $p$ and $q$ vary inversely with each other, then,
[NCERT Exemplar]
(a) $\frac{p}{q}$ remains constant.
(b) $p+q$ remains constant.
(c) $p \times q$ remains constant
(d) $p-q$ remains constant.
7. Both $x$ and $y$ vary directly with each other and when x is $10, y$ is 14 , which of the following is not a possible pair of corresponding values of $x$ and $y$ ?
(a) 25 and 35
(b) 35 and 25
(c) 35 and 49
(d) 15 and 21
8. Which of the following vary inversely with each other?
[NCERT Exemplar]
(a) speed and distance covered
(b) distance covered and taxi fare.
(c) distance travelled and time taken.
(d) speed and time taken.

| 1.d | 2.d | 3.c | 4.a | 5.a | $6, \mathrm{c}$ | $7, \mathrm{~b}$ | 8.d |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |

## I. Fill in the blanks

1. If $x=5 y$, then $x$ and $y$ vary $\qquad$ with each other.
[NCERT Exemplar]
2. When two quantities $x$ and $y$ are in proportion or vary $\qquad$ they are written as $x \alpha \frac{1}{y}$.
[NCERT Exemplar]
3. In direct proportion,,$\frac{a_{1}}{b_{1}}-\quad-\frac{a_{2}}{b_{2}}$
[NCERT Exemplar]
4. The perimeter of a circle and its diameter vary $\qquad$ with each other.
[NCERT Exemplar]
5. A car is travelling 48 km in onehour. The distance travelled by the car in 12 minutes is $\qquad$ _-
[NCERT Exemplar]

| 1. directly | 2. inverse, inversely | $3 .=$ | 4. directly | 6.9 .6 km |
| :--- | :--- | :--- | :--- | :--- |

## II. True or False

1. 6 pipes can till a tank in 24 minutes, one pipe can fill it in 144 minutes.
[NCERT Exemplar]
2. 14 workers can build a wall in 42 days. One work, can build it in 588 days.
[NCERT Exemplar]
3. When the speed Is kept fixed, time and distance vary inversely with each other.
[NCERT Exemplar]
4. Length of a side of a square and its area vary directly with each other.
[NCERT Exemplar]
5. If $x$ and $y$ are in inverse proportion, then $(x+1)$ and $(y+1)$ are also in inverse proportion.
[NCERT Exemplar]

| 1. False | 2. True | 3. False | 4. False | 5. False |
| :---: | :---: | :---: | :---: | :---: |

## I. Match the following

| Column I | Column II |
| :--- | :--- |
| 1. $x$ and $y$ vary inversely to each other | A. $\frac{x}{y}=$ Constant |
| 2. Mathematical representation of inverse variation <br> of quantities p and q | B. y will increase in proportion |
| 3. Mathematical representation of direct variation <br> of quantities m and n | C. $x y=$ Constant |
| 4. When $x=5, y=2.5$ and when $y=5, x=10$ | D. $p \alpha \frac{1}{q}$ |


| 5. When $x=10, y=5$ and when $x=20, y=2.5$ | E. $y$ will decrease in proportion |
| :--- | :--- |
| 6. $x$ and $y$ vary directly with each other | F. $x$ and $y$ directly proportional |
| 7. $x$ and $y$ vary inversely then on decreasing $x$ | G. $m \alpha n$ |
| 8. $x$ and $y$ vary directly then on decreasing $x$ | H. $x$ and $y$ vary inversely |
|  | I. $p \alpha$ |
|  | J. $m \alpha \frac{1}{2}$ |


| $1 . \mathrm{H}$ | 2.D | 3.G | $4 . \mathrm{F}$ | $5 . \mathrm{C}$ | $6 . \mathrm{A}$ | $7 . \mathrm{B}$ | 8.E |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |

## I. Very Short Answer Type Questions

1. Observe the table given below and find whether $t$ and $y$ are directly proportional :[NCERT Exemplar]

| $\mathbf{x}$ | 3 | 5 | 7 | 9 | 12 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{y}$ | 6 | 10 | 14 | 18 | 24 |

Sol. Clearly,
$\frac{x}{y}=\frac{3}{6}=\frac{5}{10}=\frac{7}{10}=\frac{9}{18}=\frac{12}{24}=\frac{1}{2}($ Constant $)$
$\therefore \mathrm{x}, \mathrm{y}$ are directly propotional.
2. A car covers 432 l , om 36 litres of petrol. How much distance world it cover in 25 litres of petrol?
[NCERT Exemplar]
Sol. Let required distance be $x \mathrm{~km}$. Then, we have

| Quantity of petrol (in litres) | 36 | 25 |
| :--- | :--- | :--- |
| Distance (in km) | 432 | x |

Clearly, less is the quantity of petrol consumed, less is the distance covered.
So, it is a case of direct proportion

$$
\therefore \quad \frac{36}{432}=\frac{25}{x} \Rightarrow \frac{1}{12}=\frac{25}{x} x=12 \times 25=300
$$

$\therefore$ Required distance is 300 km .
3. If $x$ varies inversely as $y$ and $x=400$.
[NCERT Exemplar]
Sol. Here, variation is inverse

$$
\begin{array}{ll}
\therefore \quad & 20 \times 600=400 \times \mathrm{y} \\
& y=\frac{20 \times 600}{400} \\
y=30
\end{array}
$$

4. The variable $x$ varies directly as $y$ and $x=\mathbf{8 0}$ when $y$ is 160 . What is y , when x is 64?
[NCERT Exemplar]
Sol. Given, x varies directly as m i.e., $\mathrm{x} \alpha \mathrm{m}$
Here, $\mathrm{x}_{1}=80, \mathrm{y}_{1}=160, \mathrm{x}_{2}=64, \mathrm{y}_{2}=$ ?

$$
\begin{array}{ll}
\therefore & \frac{x_{1}}{y_{1}}=\frac{x_{2}}{y_{2}} \\
& \frac{80}{60}=\frac{64}{y_{2}} \\
\Rightarrow & y_{2}=\frac{64}{80} \\
& y_{2}=128
\end{array}
$$

5. If x varies inversely as y and $\mathrm{y}=60$ and $\mathrm{x}=1.5$, Find x

Sol. Here, variation is inverse

$$
\begin{aligned}
& \therefore \quad 60 \times 1.5=4.5 \times x \\
& \quad x=\frac{60 \times 1.5}{4.5}
\end{aligned}
$$

$$
x=20
$$

6. $l$ varie directly as $m$ when $l$ is equal to 5 , where $m=\frac{2}{3}$. Find $l$, when $m=\frac{16}{3}$.
[NCERT Exemplar]
Sol. Here, $l \propto m$

$$
\begin{aligned}
& \text { Here, } l_{1}=5, m_{1}=\frac{2}{3}, l_{2}=?, m_{2}=\frac{16}{3} \\
& \therefore \\
& \frac{l_{1}}{m_{1}}=\frac{l_{2}}{m_{2}} \\
& \frac{\frac{5}{2}}{3}=\frac{l_{2}}{\frac{16}{3}} \\
& \\
& \\
& l_{2}=\frac{5 \times 3 \times 16}{2 \times 3}=40
\end{aligned}
$$

7. If x varies inversely as y when $\mathrm{x}=40$, and $\mathrm{y}=600$, then find y , when $\mathrm{x}=400$.
[NCERT Exemplar]
Sol. Here, $x$ varies inversely as $y$

$$
\begin{aligned}
& \text { Hence, } x_{1} y_{1}=x_{2} y_{2} \\
& \because \\
& x_{1}=40, y_{1}=600, x_{2}=400, y_{2}=? \\
& \\
& \\
& \Rightarrow \\
& 40 \times 600=400 \times \mathrm{y}_{2} \\
& \\
& y_{2}=\frac{40 \times 600}{400}=600
\end{aligned}
$$

## II. Very Short Answer Type Questions

1. Find the missing quantity, if $x$ varies directly as $y$,
[NCERT Exemplar]

| x | 12 | 6 |
| :---: | :---: | :---: |
| y | 48 | - |

Sol. Here, x varies directly as y ,

| x | 12 | 6 |
| :---: | :---: | :---: |
| y | 48 | a |

$$
\begin{array}{llll}
\Rightarrow & \frac{126}{48}=\frac{6}{a} & \Rightarrow & \\
\Rightarrow & a=\frac{6 \times 48}{12} & & \Rightarrow
\end{array}
$$

2. If $x$ varies inversely as $y$ and $x=4$ when $y=6$, then when $x=3$ the value of $y$ is
[NCERT Exemplar]
Sol. Here, variation is inverse

$$
\therefore \quad 4 \times 6=3 \times y \quad \Rightarrow \quad y=\frac{4 \times 6}{3}=8
$$

Hence, $y$ is $\qquad$ .

State True or False (Q. 3 \& Q. 4)
3. If $x$ and $y$ are in inverse proportion, then $(x+1)$ and $(y+1)$ are also in inverse proportion.
[NCERT Exemplar]
Sol. False.
4. If $d$ varies directly as ${ }^{t 2}$, then we can write $d^{2}=k$, where $k$ is some constant.
[NCERT Exemplar]
Sol. False.
5. Which of the following are in inverse proportion?
(i) Time taken for a fixed journey and the speed of the vehicle.
(ii) Area of cultivated land and crop harvested.

Sol. In (i) time and speed are inversly proportional because. more the speed less will be the time taken.

## I. Short Answer Type Questions

1. If two cardboard boxes occupy 500 cubic centimetres space, then how much space is required to keep 200 such boxes?
[NCERT Exemplar]
Sol. As the number of boxes increases, the space required to keep them also increases.
So, this is a case of direct proportion.

| Number of boxes | 2 | 200 |
| :--- | :--- | :--- |
| Space occupied (in cubic centimetres) | 500 | x |

$$
\begin{array}{ll}
\therefore & \frac{2}{500}=\frac{200}{x} \\
& 2 x=500 \times 200 \\
& x=\frac{500 \times 200}{2} \\
& x=50,000
\end{array}
$$

Thus, the required space is 50,000 cubic centimetres.
2. In a scout camp, there is food provision for 300 cadets for 42 days. If 50 more persons join the camp, for how many days will the provision last?
[NCERT Exemplar]
Sol. More the persons, the sooner would be the provision exhausted. So, this is a case of inverse proportion.
Let the required number of days be $x$.
Hence, $300 \times 42=(300+50) \times x$

$$
\begin{aligned}
& \frac{300 \times 42}{350}=x \\
& x=36
\end{aligned}
$$

3. Under the condition that the temperature remains constant, the volume of gas is inversely proportional to its pressure. If the volume of gas is 630 cubic centimetres at a pressure of 360 mm of mercury, then what will be the pressure of the gas if its volume is 720 cubic centimetres at the same temperature?
[NCERT Exemplar]
Sol. Given that, at contant temperature, pressure and volume of a gas are inversely proportional.
Let the required pressure be x .

| Volume of gas (in cubic centimetres) | 630 | 720 |
| :--- | :--- | :--- |
| Pressure of gas (in mm). | 360 | x |

Then,

$$
630 \times 360=720 \times x
$$

$$
\frac{632 \times 360}{720}=x
$$

$\therefore \quad x=315$
Therefore, the required pressure is 315 mm of mercury.
4. 44 cows can graze a field in 9 days. How many less/ more cows will graze the same field in 12 days?
[NCERT Exemplar]
Sol. Let the number of cow graze the field be $x$.

| Number of Cows | 44 | X |
| :--- | :---: | :---: |
| Number of days | 9 | 12 |

Clearly, number of cows will be number of days.
So, it is a case of inverse proportion.
i.e.

$$
\begin{aligned}
& x_{1} y_{1}=x_{2} y_{2} \\
& 44 \times 9=x \times 12 \\
& x=\frac{44 \times 9}{12}=33
\end{aligned}
$$

So, difference $=44-33=11$
In 12 days, 11 less cow will be required.
5. The mass of an aluminium rod varies directly with its length. If a 16 cm long rod has a mass of 192 g , find the length of the rod whose mass is 105 g .
[NCERT Exemplar]
Sol. Here, mass of rod $\propto$ length of the rod

Let man and length of the rod be $m$ in and $l$, respectively.

$$
\begin{array}{ll}
\because & m \propto l \\
\Rightarrow & \frac{m_{1}}{l_{1}}=\frac{m_{2}}{l_{2}} \\
& \mathrm{~m}_{1}=192 \mathrm{~g}, \mathrm{l}_{1}=16 \mathrm{~cm} \\
& \mathrm{~m}_{2}=105 \mathrm{~g} \text { and } \mathrm{l}_{2}=? \\
\therefore \quad & \frac{192}{6}=\frac{105}{l} \\
& \mathrm{l}_{2}=\frac{16 \times 105}{192} \\
& \mathrm{l}_{2}=8.75 \mathrm{~cm}
\end{array}
$$

6. The cost of 27 kg of iron is Rs. 1,080 , what will be the cost of 120 kg of iron of the same quality?
[NCERT Exemplar]
Sol. Let the cost of iron be $x$.

| Weight of iron | 27 | 120 |
| :--- | :--- | :--- |
| Cost of iron | 1080 | x |

As the cost will increase the weight. Hence, it is directly proportional.

$$
\begin{array}{ll}
\text { So, } & \frac{27}{1080}=\frac{120}{x} \\
& x=\frac{1080 \times 120}{27} \\
& x=4800
\end{array}
$$

7. If 50 metres of a cloth costs Rs.3725, how much cloth can be purchased for Rs.1788?
[NCERT Exemplar]
Sol. Let the length of cloth $=x$ metres, then

| Length of cloth (in metres) | 50 | x |
| :--- | :---: | :---: |
| Cost (in rupees) | 3725 | 1788 |

Clearly, less the cost, less the length of cloth.
Then by direct proportion,

$$
\begin{array}{ll} 
& \frac{50}{3725}=\frac{x}{1788} \\
\Rightarrow & \frac{2}{149}=\frac{x}{1788} \\
\Rightarrow & x \times 149=1788 \times 2 \\
\Rightarrow & x=\frac{1788 \times 2}{149}=24
\end{array}
$$

Hence, the required length of cloth $=24 \mathrm{~cm}$.
8. In a hostel of 50 girls, there are food provisions for 40 days. If 30 more girls join the hostel, how long will these provision last?
Sol. $\quad$ Number of gives added $=30$
$\therefore$ Now total number of girls $=50+30=80$
For more number of girls, the food his last less number of days.
$\therefore$ It is a case of inverse proportion

$$
\begin{aligned}
50 \times 40 & =80 \times x \\
x & =\frac{50 \times 40}{80}=25
\end{aligned}
$$

Therefore, the food will now last for 25 days.

## II. Short Answer Type Questions

1. It is given that 1 varies directly as $m$.
(i) Write an equation which relates $l$ and $m$.
(ii) Find $l$, when $m$ is 33 and $k$ (constant of proportion) is $\frac{1}{3}$.

Sol. (i) $\frac{l}{m}=k, \mathrm{k}$ is a constant
(ii) Given, $\mathrm{m}=33 ; k=\frac{1}{3}$
$\Rightarrow \quad \frac{l}{m}=k \quad \Rightarrow \quad \frac{l}{33}=\frac{1}{3} \quad \Rightarrow \quad 3 l=33$
$\Rightarrow \quad l=11$
2. If 25 metres of cloth costs Rs337.50, then what will be the length of the cloth bough. for 1810 ?
[NCERT Exemplar]
Sol. Let the length of cloth be $x$ metres.
We form table as shown below:

| Length of cloth (metres) | 25 | $x$ |
| :--- | :---: | :---: |
| Cost of cloth (Rs) | 337.5 | 810 |

As the cost will increase by increasing the length. Hence. it is directly proportional.
So, $\frac{x_{1}}{y_{1}}=\frac{x_{2}}{y_{2}}$
We have, $\frac{25}{337.5}=\frac{x}{810}$
$\Rightarrow \quad x=\frac{25 \times 810}{337.5} \quad \Rightarrow x=60$ metres
3. 44 cows can graze a field in 9 days. How many less/more cows willgraze the same field in 12 days?
[NCERT Exemplar]
Sol. Let the required number of cows be $x$.
We form a table as shown below:

| Length of cloth (metres) | 44 | $x$ |
| :--- | :---: | :---: |
| Cost of cloth (Rs) | 9 | 12 |

It is inverse proportion as less the number of cows grazing the field, More the days.
i.e.,

$$
\mathrm{x}_{1} \mathrm{y}_{1}=\mathrm{x}_{2} \mathrm{y}_{2}
$$

We have $44 \times 9=x \times 12$

$$
x=\frac{44 \times 9}{12}=3
$$

Therefore, (44-33) i.e., 11 less cows will graze the field in 12 days
4. The following table shows the distance travelled by one of the new eco-friendly energy efficient cars travelled on gas.
[NCERT Exemplar]

| Litres of gas | 1 | 0.5 | 2 | 2.5 | 3 | 5 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Distance $(\mathrm{km})$ | 15 | 7.5 | 30 | 37.5 | 45 | 75 |

Which type of properties are indicated by the table? How much distance will be covered by the car in 8 litres of gas?

Sol. The above table is given as

| Litres of gas | 1 | 0.5 | 2 | 2.5 | 3 | 5 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Distance $(\mathrm{km})$ | 15 | 7.5 | 30 | 37.5 | 45 | 75 |

(i)

$$
\begin{array}{llll}
\text { Let } & \mathrm{x}_{1}=1 ; & \mathrm{x}_{2}=0.5 ; & \mathrm{x}_{3}=2 \\
& \mathrm{y}_{1}=15 ; & \mathrm{y}_{2}=75 ; & \mathrm{y}_{3}=30 \\
\Rightarrow & \frac{\mathrm{x}_{1}}{\mathrm{y}_{1}}=\frac{1}{15}, \frac{x_{2}}{y_{2}}=\frac{0.5}{7.5}=\frac{1}{15}, \frac{x_{3}}{y_{3}}=\frac{2}{30}=\frac{1}{30} &
\end{array}
$$

All the above $x$ and $y$ are in direct proportion
(ii) Let distance covered by car in 8 litres of gas be $x \mathrm{~km}$.

| Litres of gas | 1 | 8 |
| :--- | :---: | :---: |
| Distance $(\mathrm{km})$ | 15 | $x$ |

As it is in direct proportion

$$
\therefore \quad \frac{\mathrm{x}_{1}}{\mathrm{y}_{1}}=\frac{x_{2}}{y_{2}} \quad \Rightarrow \quad \frac{1}{15}=\frac{8}{x}
$$

5. A school has 9 periods a day each of 50 minute duration. How many periods will there be if the duration of every period is reduced by 5 minute?

Sol. Present duration of period $=(50-5)$ minute $=45$ minute.
Let the number of period be ' $x$ '.

| Number of periods | 9 | $x$ |
| :--- | :---: | :---: |
| Duration (in minute) | 15 | $x$ |

Since, less the duration of period, more the number of periods.
$\therefore \quad$ It is an inverse proportion, i.e., $\mathrm{x}_{1} \mathrm{y}_{1}=\mathrm{x}_{2} \mathrm{y}_{2}$
We have

$$
\begin{aligned}
\Rightarrow \quad & 9 \times 50=x \times 45 \\
& \mathrm{x}=\frac{9 \times 50}{45} \Rightarrow \quad \mathrm{x}=10
\end{aligned}
$$

Thus, number of periods is 10 .
6. 6 pipes are required to fill a tank in 1 hour 20 minutes. How long will it take if only 5 pipes of the same type are used?

Sol. Let the desired time to fill the tank be x minutes. Thus, we have the fallowing table.

| Number of pipes | 6 | 5 |
| :--- | :---: | :---: |
| Time (in minutes) | 80 | x |

Lesser the number of pipes. more will be the time required by it to till the tank. So. this is a case of inverse proportion.

Hence. $80 \times 6=x \times 5$

$$
\left[\mathrm{x}_{1} \mathrm{y}_{1}=\mathrm{x}_{2} \mathrm{y}_{2}\right]
$$

or $\quad \frac{80 \times 6}{5}=x$
or $\quad x=96$
Thus. time taken to fill the tank by 5 pipes is 96 minutes or 1 hour 36 minutes.

## 7. Find value of $x$ and $y$ if $a$ and $b$ are in inverse proportion:

| (a) | 12 | $x$ | 8 |
| :---: | :---: | :---: | :---: |
| (b) | 30 | 5 | $y$ |

Sol. Given $a$ and $b$ are in inverse proportion.

| (a) | 12 | $x$ | 8 |
| :---: | :---: | :---: | :---: |
| (b) | 30 | 5 | $y$ |

$$
\begin{array}{llc}
\text { Here, } \mathrm{x}_{1}=12, \quad \mathrm{x}_{2}=\mathrm{x}, & \mathrm{y}_{1}=30, & \mathrm{y}_{2}=5 \\
\therefore & \mathrm{x}_{1} \mathrm{y}_{1}=\mathrm{x}_{2} \mathrm{y}_{2} \\
\Rightarrow & 12 \times 30=5 \times x & \\
& \mathrm{x}=\frac{12 \times 30}{5}=72 \quad \Rightarrow & x=72 \\
\text { And, } & \mathrm{x}_{2} \mathrm{y} 2=\mathrm{x}_{3} \mathrm{y}_{3} & \\
\text { here } & \mathrm{x} 2=72 & \\
& \mathrm{y} 2=5 & \\
72 \times 5=8 \\
& \\
& \\
& \mathrm{y}=\frac{725}{8}=45 & \Rightarrow y=45
\end{array}
$$

8. The scale of a map is given as I : $3,00,00.000$. Two cities are 4 cm apart on the maps. Find the actual distance between them.

Sol. Let the map distance be $x \mathrm{~cm}$ and actual distance by $y \mathrm{~cm}$, then

$$
1: 30000000=x: y
$$

or

$$
\frac{1}{30 \times 10}=\frac{x}{y}
$$

Since $x=4$ so, $\frac{1}{3 \times 10^{7}}=\frac{4}{y}$
or $\quad y=4 \times 3 \times 10^{7}=12 \times 10^{7} \mathrm{~cm}=1,200 \mathrm{~km}$.
Thus, two cities, which are 4 cm apart on the map, are actually 1200 km away from each other.

## I. Long Answer Type Questions.

1. A volleyball court is in a rectangular shape and its dimensions are directly proportional to the dimensions of the swimming pool given below. Find the width of the pool.
[NCERT Exemplar]


Sol. Here, a volleyball court and a swimming pool both are in rectangular shape and their dimensions are directly proportional. Let $l$ and $b$ are length and breadth, respectively.

$$
\begin{array}{ll}
\text { If } & l=b \\
\Rightarrow & \frac{l_{1}}{b_{1}}=\frac{l_{2}}{b_{2}}
\end{array}
$$

Here, $, \mathrm{l}_{1}=18 \mathrm{~m}, \mathrm{~b}_{2}=9 \mathrm{~m}, \mathrm{l}_{2}=75 \mathrm{~m}, \mathrm{~b}_{2}=$ ?
Then, $\quad \frac{8}{9}=\frac{75}{b_{2}}$
$2 b_{2}=75$
$\mathrm{b}_{2}=\frac{75}{2}$
$\mathrm{b} 2=37.5 . \mathrm{m}$
So, the width of the pool is 37.5 m .
2. A water tank casts a shadow 21 m long. A tree of height 9.5 m casts a shadow 8 m long at the same time. The lengths of the shadows are directly proportional to their heights. Find the height of the tank.
[NCERT Exemplar]


Sol. Here, the length of the shadow are directly proportional to their length.
Here, $\mathrm{h}_{1}=9.5 \mathrm{~m}, \mathrm{l}_{1}=8 \mathrm{~m}, \mathrm{~h}_{2}=\mathrm{x}, \mathrm{l}_{2}=21 \mathrm{~m}$

$$
\begin{aligned}
& \frac{h_{1}}{l_{1}}=\frac{h_{2}}{l_{2}} \\
& \frac{9.5}{8}=\frac{x}{21} \\
& x=\frac{9.5 \times 21}{8}=24.9 \mathrm{~m}
\end{aligned}
$$

$\therefore \quad$ The height of the tank is 24.9.
3. The students of Anju's class sold posters to raise money. Anju wanted to create a ratio for finding the amount of money her class would make for different numbers of posters sold. She knew they could raise Rs. 250 for every, 60 posters sold.
[NCERT Exemplar]
a. How much money would Anju's class make for selling 102 posters ?
b. Could Anju's class raise exactly? 2,000? If so, how many posters would they need to sell? If not, why?

Sol. Here, Aniu's class raised tor every 60 posters.
a. Let A be the money.

| Money Raised | 250 | x |
| :--- | :---: | :---: |
| Number of posters sold | 60 | 102 |

Clearly, number of poster will increase, money will also increased.

$$
\begin{array}{ll}
\therefore & \frac{250}{60}=\frac{x}{102} \\
& x=\frac{102 \times 250}{60}=425
\end{array}
$$

Money raised by 102 poster is Rs. 425 .
b. No, could not raise exact, Rs.2000,

Let the number of poster Anju, class need to sell be $y$.

| Money Raised (in Rs) | 250 | 2000 |
| :--- | :---: | :---: |
| No of posters sold | 60 | $n$ |

$$
\begin{array}{ll}
\therefore \quad & \frac{250}{60}=\frac{2000}{n} \\
& n=\frac{2000 \times 60}{250}=480
\end{array}
$$

So, required number of posters are 480.
4. If 25 metres of costs Rs. 337.50 , then
a. What will be the cost of 40 metres of the same type of cloth?
b. What will be the length of the cloth bought for Rs.810?
[NCERT Exemplar]
Sol. a. I.et cost of cloth be Rs.x.

| Length of cloth (m) | 25 | 40 |
| :--- | :---: | :---: |
| Cost of cloth (?) | 337.5 | x |

Here, cost will be increase by increasing the length of cloth. Hence, it is directly proportional.

$$
\begin{aligned}
& \frac{25}{337.5}=\frac{40}{x} \\
& x=\frac{337.5 \times 40}{25}=R s .540
\end{aligned}
$$

b. Let the length of cloth be y m .

| Length of cloth (m) | 25 | y |
| :--- | :---: | :---: |
| Cost of cloth (?) | 337.5 | 810 |

Here, as the length will increase by increasing the amount So, it is directly proportional.

$$
\begin{array}{ll} 
& \frac{25}{337.5}=\frac{y}{810} \\
\Rightarrow & y=\frac{25 \times 810}{337.5} \\
\Rightarrow & y=60 \text { metres. }
\end{array}
$$

5. Observe the following table, where x and y are in inverse variation :
[NCERT Exemplar]

| $\mathbf{x}$ | (i) $p_{1}$ | 200 | 300 |
| :--- | :--- | :--- | :--- |
| $\mathbf{y}$ | 60 | 30 | (ii) $\mathrm{p}_{2}$ |

Find the unknown quantities.
Sol. Let $x_{1}=p_{1}, x_{2}=200, x_{3}=300$
and $y_{1}=60, y_{2}=30, y_{3}=p_{2}$
Since, $x$ and $y$ are in inverse variation.
a. $\therefore$
$\mathrm{x}_{1} \mathrm{y}_{1}=\mathrm{x}_{2} \mathrm{y}_{2}$
$\Rightarrow \quad \mathrm{p}_{1} \times 60=200 \times 30$

$$
\therefore \quad \mathrm{p}_{1}=\frac{200 \times 30}{60}=10
$$

b. Also,

$$
\mathrm{x}_{2} \mathrm{y}_{2}=\mathrm{x}_{3} \mathrm{y}_{3}
$$

$$
\Rightarrow \quad 200 \times 30=300 \times p_{2}
$$

$$
\therefore \quad \mathrm{p}_{2}=\frac{200 \times 30}{300}=20
$$

6. It is given thatl varies directly as m .
a. Write an equation which relates $l$ and $m$.
b. Find the constant of proportion $(\mathrm{k})$, when $l$ is 6 then $m$ is 18 .
c. Find $l$, when or is 33 .
d. Find $m$ when $l$ is 18 .
[NCERT Exemplar]
a. $l=\mathrm{km}$, where $k$ is a constant
b.

$$
\begin{array}{rlrl} 
& l & =6, m=18 \\
& l & =k m \\
& 6 & =k \times 18 \\
\Rightarrow & & \mathrm{k} & =6 / 18 \\
\therefore & & k & =\frac{1}{3}
\end{array}
$$

$\begin{array}{ll}c . & m=33, l=? \\ \because \quad l & =k m \\ l=\frac{1}{3} \times 33\end{array}$

$$
\left[\because k=\frac{1}{3}\right]
$$

$$
\begin{aligned}
l & =11 \\
l & =8, m=? \\
l & =k m \\
\Rightarrow \quad 8 & =\frac{1}{3} \times m \\
m & =24
\end{aligned}
$$

d.

## II. Long Answer Type Questions.

1. A and B can harvest a field in 30 days. A alone can do $\frac{1}{5}$ of this work in 10 days. In how many days can $B$ alone harvest the field?

Sol. $\because \quad$ Time taken by A and B together to do harvesting work $=30$ days
$\because \quad$ The work clone by both A and B in 1 day $=\left(\frac{1}{30}\right)^{t h}$ part of this work
Again,
$\because \quad$ Time taken by A alone to do $\frac{1}{5}$ of this work $=10$ days
$\because \quad$ Time taken by A alone to do I work (complete work) $=10 \times 5$ days $=50$ days
$\because \quad$ The work done by A alone in 1 day $=\left(\frac{1}{50}\right)^{\text {th }}$ part of this work
$\because \quad$ The work done by $B$ alone in 1 day
$=$ Work done by boils in one day - Work done by A alone in one clay
$=\left(\frac{1}{30}\right)^{t h}$ part of the work $-\left(\frac{1}{50}\right)^{t h}$ part of the work
$=\left(\frac{1}{30}-\frac{1}{50}\right)^{t h}$ part of the work $=\left(\frac{5-3}{150}\right)^{t h} \mathrm{r}$ part of the work
$=\left(\frac{2}{150}\right)^{t h}$ part of the work $=\left(\frac{1}{75}\right)^{t h}$ part of the work
So, B alone can harvest the field in $\frac{75}{1}$ days $=75$ days.
2. Here is a key board of harmonium:
(i) Find the ratio of white keys to black keys on the keyboard.

(ii) What is the ratio of black keys to all keys on the given keyboard?
(iii) This pattern of keys is repeated on larger keyboard. How many black keys would you expect to find on a keyboard with 14 such patterns?
[NCERT Exemplar]
Sol.
(i) $10: 7$
(ii) $7: 17$
(iii) Let the required number of black keys be $x$.

As the size increases, number of keys will increase. It is in direct proportion.

$$
\begin{array}{ll}
\therefore & \frac{7}{17}=\frac{x}{238} \\
\therefore & 238 \times 7=17 \times x \\
& x=\frac{238 \times 7}{17}=98
\end{array}
$$

Required number of black keys $=98$.
3. The students of Anju's class sold posters to raise money. Anju wanted to create a ratio for finding the amount of money her class would make for different numbers, of posters sold. She knew they could raise Rs. 250 for every 60 posters sold.
(i) How much money would Anju's class make for selling 102 posters?
(ii) Could Anju's class raise exactly Rs.2,000? If so, how many posters would they need to sell?
[NCERT Exemplar]
Sol. We are given,
Anju's class raised Rs250 for every 60 posters.
(i) Let $x$ be the money Anju's class will make by 102 posters.

Table shown under

| Money Raised (in Rs) | 250 | $x$ |
| :--- | :---: | :---: |
| Number of Posters sold | 60 | 102 |

As, number of posters will increase, money raised will also increase.
So, it is direct proportion.

$$
\begin{array}{ll}
\therefore & \frac{250}{60}=\frac{x}{102} \\
\Rightarrow & x=\frac{102 \times 250}{60}=425
\end{array}
$$

Money raised by 102 posters is Rs. 425 .
(ii) Let the number of posters they used to sell to raise Rs. 2000 be n

$$
\begin{aligned}
\therefore \quad & \frac{250}{60}=\frac{2000}{n} \\
& n=\frac{2000 \times 60}{250}=480
\end{aligned}
$$

Required number of poster are 480.
4. Inlet A can fill a water tank in 8 hours and inlet B can fill it in 10 hours. Both Aand Bare opened and after two hours, inlet $A$ is closed. How much me will $B$ take to fill the remaining part of the tank?
Sol. Given,
Time taken by inlet A to fill the tank $=8$ hours (h)
Time taken by inlet B to fill the tank $=10$ hours
Therefore,
the part of tank filled up by inlet A in 1 hour $=\left(\frac{1}{8}\right)^{\text {th }}$ part of the tank
the part of tank filled tip by inlet B in 1 in 1 hour $=\left(\frac{1}{10}\right)^{\text {th }}$ part of the tank

If they open together, then
the part of tank filled up by both in 1 hour $=\left(\frac{1}{8}+\frac{1}{10}\right)^{t h}$ part of the tank

$$
\begin{aligned}
& =\left(\frac{5+4}{40}\right)^{t h} \text { art of the tank th } \\
& =\left(\frac{9}{40}\right)^{t h} \text { part of the tank }
\end{aligned}
$$

The part of tank filled up by both in 2 hours $=\left(\frac{9}{40} \times 2\right)^{\text {th }}$ part of the tank

$$
=\left(\frac{9}{20}\right)^{t h} \text { part of the tank }
$$

$\because \quad$ Remaining part of tank to be filled by B alone $=\left(1-\frac{9}{20}\right)^{\text {th }}$ part of the tank

$$
\begin{aligned}
& =\left(\frac{20-9}{20}\right)^{t h} \text { part of the tank } \\
& =\left(\frac{11}{20}\right)^{t h} \text { part of the tank }
\end{aligned}
$$

$\because \quad\left(\frac{1}{10}\right)^{\text {th }}$ part of tank is filled by B in 1 hour
$\because \quad$ Complete tank is filled by B in 10 hours.
$\therefore \quad\left(\frac{11}{20}\right)^{\text {th }}$ part of the tank is filled by B in $\frac{11}{20} \times 10$ hours $=\frac{11}{2}$ hours $=5 \frac{1}{2}$ hours
5. Sita can do a piece of work alone in 3 days, Rita can do it alone in 6 days and Gita in 9 days. If all three work together, in how many days can they complete the work?
Sol. Given,
Time taken by Sita to do the work $=3$ days
Time taken by Rita to do the work $=6$ days
Time taken by Gita to do the work $=9$ days
Therefore,
work done by Sita in 1 day $=\left(\frac{1}{3}\right)^{\text {th }}$ part of the work
work done by Rita in 1 day $=\left(\frac{1}{6}\right)^{\text {th }}$ part of the work
work done by Gita in 1 day $=\left(\frac{1}{9}\right)^{\text {th }}$ - part of the work
If they work together. then
work done by all the three in 1 day $=\left(\frac{1}{3}+\frac{1}{6}+\frac{1}{9}\right)^{t h}$ part of the work.
$=\left(\frac{6+3+2}{18}\right)^{\text {th }}$ part of the work
$=\left(\frac{11}{18}\right)^{t h}$ part of the work
So, Sita, Rita and Gita can do the complete work in $\frac{8}{11}$ days $=1 \frac{7}{11}$ days

## I. High Order Thinking Skills [HOTS] Questions

an

1. A private taxi charges a fare of Rs. 260 for a journey of 200 km . How much would it travel for Rs. 279.50
[NCERT Exemplar]
Sol. Let the distance travelled by a taxi $=x \mathrm{~km}$, then

| Fare (in Rupees) | 260 | 279.50 |
| :--- | :---: | :---: |
| Distance (in km) | 200 | x |

Clearly, the taxi will travel more for more money, then by direct variation.

$$
\begin{array}{ll} 
& \frac{260}{279.50}=\frac{200}{x} \\
\Rightarrow & x=\frac{200 \times 279.50}{260} \\
\Rightarrow & x=215
\end{array}
$$

Hence, the taxi will travel $=215 \mathrm{~km}$
2. (a) In a fort, 300 men had provisions for 90 days. After 20 days, 50 men left the fort. How long would the food last at the same rate?

Sol. (a) Since, the remaining days $=(90-20)$ days $=70$ days
But remaining number of men $=300-50=250$
Let the number of days $=x$, then

| Number of men | 300 | 250 |
| :--- | :---: | :---: |
| Number of days | 70 | x |

Clearly less men will have food for more days. Then, by inverse variation,

$$
\begin{aligned}
& \frac{300}{250}=\frac{x}{70} \\
& \text { or } \\
& \text { or } \\
& \text { or }
\end{aligned} \quad x=\frac{x=70=x \times 250}{250}=84
$$

Hence. the required number of days $=84$.

## II. High Order Thinking Skills [HOTS] Questions

I. The area of a square increases if we increase the length of its sides. Are the two quantities in direct proportion? Give reason.
Sol. No, the two quantities are not in direct proportion because, it we in, cease the length of its sides, the ratio of the two quantities (area. length) is not constant. So it is not in direct proportion.
2. If we fix time period and the rate of interest, simple interest changes proportionally with principal.

Would there be a similar relationship for compound interest? Why?
Sol. For simple interest.
$\mathrm{SI} \frac{\mathrm{P} \times \mathrm{r} \times \mathrm{t}}{100} \Rightarrow \frac{S I}{P}=\frac{r . t}{100}$
Since rate and time period are constant, then $S /$ changes directly according to $P$.
Thus, simple interest changes in direct proportion with principal.

For compound interest,
$\mathrm{CI}=\mathrm{P}\left\{\left(1+\frac{\mathrm{r}}{100}\right)^{t}-1\right\}$
$\mathrm{CI}=\mathrm{P}\left\{\frac{\left(1+\frac{\mathrm{r}}{100}\right)^{t}-1}{\mathrm{P}}\right\}=\left\{\left(1+\frac{\mathrm{r}}{100}\right)^{t}-1\right\}$
Siam rate and time period are constant then $\left\{\left(1+\frac{\mathrm{r}}{100}\right)^{t}-1\right\}$ is constant.
i.e., CI changes with p

Thus, the compound interest also changes directly in proportion with principal.
3. In 8 days, the earth picks up $6.4 \times 10^{7} \mathrm{~kg}$ of dust from the atmosphere. How much dust will it pick up in 15 days?

Sol. Number of days $=8$
Quantity of dust picked up $=6.4 \times 10^{7} \mathrm{~kg}$
If number of days $=15$
Then let quantity of dust picked up $=x \mathrm{~kg}$
As the number of days varies directly to the quantity of dust picked up. This is a case of direct proportion.

| Number of days | 8 | 15 |
| :--- | :---: | :---: |
| Quantity of dust $(\mathrm{kg})$ | $6.4 \times 10^{7}$ | $x$ |

So, $\quad \frac{8}{6.4 \times 10^{7}}=\frac{15}{x} \quad$ or $\quad \frac{x}{15}=\frac{6.4 \times 10^{7}}{8}$
or, $\quad x=\frac{6.4 \times 10^{7}}{8} \times 15=12 \times 10^{7} \mathrm{~kg}$
Quantity of dust picked up $=12 \times 10^{7} \mathrm{~kg}$
4. Manish travels to his office by a ear at a speed of $50 \mathrm{~km} / \mathrm{h}$ and reaches office 10 minutes late. If he drives his car at a speed of $60 \mathrm{~km} / \mathrm{h}$, he reaches 6 minutes early. What is the distance of his office from his home?
Sol. Let the distance of office from his !ionic be x km .
Case 1: When the speed of car is $50 \mathrm{~km} / \mathrm{h}$.
Time taken $=\frac{\text { Distance }}{\text { Speed }}=\frac{x}{50}$ hour $=\left(\frac{x}{50} \times 60\right)$ minuts $=\frac{6 x}{5}$ minutes
Case 2: When the speed of car is $60 \mathrm{~km} / \mathrm{h}$.
Time taken $=\frac{\text { Distance }}{\text { Speed }}=\frac{x}{60}$ hour $=\left(\frac{x}{60} \times 60\right)$ minuts $=x$ minutes
Difference of time taken in both cases $=\left(\frac{6 x}{5}-x\right)$ minutes

$$
=\left(\frac{6 \mathrm{x}-5 \mathrm{x}}{5}\right) \text { minutes }=\left(\frac{x}{5}\right) \text { minutes }
$$

But difference in time is given $10+6=16$ minutes.
$\therefore \quad\left(\frac{x}{5}\right)$ minutes $=16$ minutes

$$
\Rightarrow \quad \frac{x}{5}=16 \quad \Rightarrow \quad x=16 \times 5 \quad \Rightarrow \quad x=80 \mathrm{~km}
$$

Hence, distance from office to home is 80 km .

## I. Value Based Questions

a. If 56 men can do a piece of work in 42 days. How many men will do it in 14 days?
b. If $x$ and $y$ vary inversely as each other and $x=10$ when $y=6$, Find $y$, when $x=15$.
[NCERT Exemplar]
Sol. a. Let the number of men $=x$

| Number of men | 56 | x |
| :--- | :--- | :--- |
| Number of days | 42 | 14 |

Here, more the number of men and less the number of days, then
$\frac{56}{x}=\frac{14}{42}$
or $\quad 14 \times x=42 \times 56$
or $\quad x=\frac{42 \times 56}{14}$
or $\quad x=168$
Hence, number of men $=168$.
b. According to problem

$$
\begin{aligned}
& 10 \times 6=15 \times y \\
\text { or } & y=\frac{10 \times 6}{15} \\
\text { or } & y=4
\end{aligned}
$$

2. 

| a | 4 | 7 | 21 |
| :---: | :---: | :---: | :---: |
| $\mathbf{b}$ | 12 | 21 | 63 |

From above table in which a and $b$ very directly. Find the constant of variation if $a$ and $b$ are in direct variation.

Sol. We have,

$$
\begin{array}{ll} 
& \frac{4}{12}=\frac{7}{21}=\frac{21}{63} \\
\text { or } & \frac{1}{3}=\frac{1}{3}=\frac{1}{3}
\end{array}
$$

Hence, constant variation $=\frac{1}{3}$.

