Grade IX

Lesson :1 NUMBER SYSTEM

Objective Type Questions

I. Multiple choice questions

1. A rational number equivalent to $\frac{5}{7}$ is

   a) $\frac{15}{17}$ b) $\frac{25}{27}$ c) $\frac{10}{14}$ d) $\frac{10}{27}$

   Sol. c

2. An example of a whole number is

   a) 0 b) $-\frac{1}{2}$ c) $\frac{11}{5}$ d) -7

   Sol. a

3. Given a rational number $-\frac{5}{9}$.This rational number can also be known as

   a) a natural number b) a whole number c) an integer d) a real number

   Sol. d

4. The rational number $0.\overline{3}$can also be written as

   a) 0.3 b) $\frac{3}{10}$ c) 0.33 d) $\frac{1}{3}$

   Sol. d

5. If the decimal representation of a number is non-terminating non-recurring, then the number is

   a) a natural number b) a rational number

   c) a whole number d) an irrational number

   Sol. d

6. A rational number between $\frac{1}{7}$ *and* $\frac{2}{7}$ *is*

a) $\frac{1}{14}$ b) $\frac{2}{21}$ c) $\frac{5}{14}$ d) $\frac{5}{21}$

Sol : $\frac{1}{7} = \frac{1}{7} \times \frac{3}{3} = \frac{3}{21}$; $\frac{2}{7} = \frac{2}{7} \times \frac{3}{3} = \frac{6}{21}$

$$\Rightarrow A\ rational\ number\ between\ \frac{1}{7}\ and\ \frac{2}{7}\ is\ \frac{5}{21}$$

∴ Correct answer is (d).

7. The number 1.101001000100001…… is

a) a natural number b) a whole number

c) a rational number d) an irrational number

Sol. d

8. Irrational number between 1.011243….and 1.012243….is

a) 1.011143…. b) 1.012343…. c) 1.01152243…. d) 1.013243

Sol. c

9. Every point on a number line

a) can be associated with a rational number

b) can be associated with an irrational number

c) can be associated with a natural number

d) can be associated with a real number.

Sol. d

10. In meteorological department, temperature is measures as a

a) natural number b) whole number c) rational number d) irrational number

Sol. c

11. The number of irrational numbers between 15 and 18 is infinite.

a) True b) false

Sol. a

12. Write a rational number between rational numbers $\frac{1}{9}$ and $\frac{2}{9}$

Sol. A rational number between $\frac{1}{9}$ and $\frac{2}{9}$ $is = \frac{\frac{1}{9}+\frac{2}{9}}{2} = \frac{3}{9x2} = \frac{1}{6}$

13. Write a rational number not lying between $-\frac{1}{5}$ and $-\frac{2}{5}$

Sol. $-\frac{3}{5}$

14. Write $\frac{327}{500}$ in decimal form.

Sol. $\frac{327}{500} = 0.654$

15. Write a rational number which does not lie between the rational numbers $-\frac{2}{3}$ and $-\frac{1}{5}$

Sol. $\frac{3}{10}$

16. Write two irrational numbers

Sol. $\sqrt{7}, \sqrt{11}, \sqrt{12}, \sqrt{14}\,.etc$ (any two)

II. Multiple choice questions

1. The product of any two irrational number is

(a) always an irrational number (b)always a rational number

(c) always an integer (d) sometimes rational, sometimes irrational

Sol. d

2. On adding $2\sqrt{3} + 3\sqrt{2}$, we get

(a) $5\sqrt{3}$ (b) 5 $(\sqrt{3} + \sqrt{2})$ (c) $2\sqrt{3} + 3\sqrt{2}$ (d) None of these

Sol. c

3. On Dividing $6\sqrt{27}$ by $2\sqrt{3}$, we get

(a) $3\sqrt{9}$ (b) 6 (c) 9 (d) None of these

$Sol: \frac{6\sqrt{27}}{2\sqrt{3}} = \frac{3\times 3\sqrt{3}}{\sqrt{3}} = 9$

Sol. c

4. $2 - \sqrt{7}$ is

(a) a rational number (b) an irrational number

(c) an integer (d) a natural number

Sol. b

5. $(-3 + 2\sqrt{3} - \sqrt{3})$ is

(a) an irrational number (b) a positive rational number

(c) a negative rational number (d) no integer

Sol. a

6. $(\sqrt{12} + \sqrt{10} - \sqrt{2})$is

(a) a positive rational number (b) equal to zero

(c) an irrational number (d) a negative integer

Sol. c

7. $2\sqrt{3} + \sqrt{3}$, is equal to

(a) $2\sqrt{6}$ (b) 6 (c) $3\sqrt{3}$ (d) $4\sqrt{6}$

Sol. c

8. On simplifying $(\sqrt{5} + \sqrt{7})^2$, *we get*,

(a) 12 (b) $\sqrt{35}$ (c) $\sqrt{5} + \sqrt{7}$ (d) $12 + 2\sqrt{35}$

Sol: $(\sqrt{5} + \sqrt{7})^2 = (\sqrt{5})^2 + (\sqrt{7})^2 + 2\sqrt{5}.\sqrt{7}$

$= 5 + 7 + 2\sqrt{35} = 12 + 2\sqrt{35}$

Sol. d

9. For rationalizing the denominator of the expression $\frac{1}{\sqrt{12}}$, we multiply and divide by

(a) $\sqrt{6}$ (b) 12 (c) $\sqrt{2}$ (d) $\sqrt{3}$

Sol. d

10. To rationalize the denominator of the expression $\frac{1}{\sqrt{7}-\sqrt{6}}$, we multiply and divide by

(a) $\sqrt{7} + \sqrt{6}$ (b) $\sqrt{6}$ (c) $\sqrt{7}.\sqrt{6}$ (d) $\sqrt{7}$

Sol. a

11. $\sqrt{10} \times \sqrt{15}$ is equal to

(a) $6\sqrt{5}$ (b) $5\sqrt{6}$ (c) $\sqrt{25}$ (d) $10\sqrt{5}$

Sol : $\sqrt{10} \times \sqrt{15} = \sqrt{2} \times \sqrt{5}$ X $\sqrt{3} \times \sqrt{5} = 5\sqrt{6}$

Sol. b

12. $-\frac{\sqrt{28}}{\sqrt{343}}$ is

(a) a natural number (b) a fraction

(c) an irrational number (d) a rational number

Sol : $\frac{-\sqrt{28}}{\sqrt{343}} = \frac{-2\sqrt{7}}{7\sqrt{7}} = -\frac{2}{7}$

Which is of the form $\frac{p}{q}$, where p and q are integers and q$\neq 0$.

So, $\frac{-\sqrt{28}}{\sqrt{343}}$ is a rational number.

Sol.d

13. After rationalizing the denominator of $\frac{5}{3\sqrt{2}-2\sqrt{3}}$, we get denominator as 7.

a. True b. False

Sol. b

14. Addition of expression $5\sqrt{3} - 2\sqrt{7}$ and $2\sqrt{3}$ + $\sqrt{5}$is

a.$5\sqrt{3} - 2\sqrt{7} + 2\sqrt{3}$ - $\sqrt{5}$ b.$10\sqrt{6} - 2\sqrt{7} + \sqrt{5}$

c.$7\sqrt{3} - 2\sqrt{7} + \sqrt{5}$ d.*none of these*

Sol. c

15. The value of $\frac{\sqrt{32}+\sqrt{48}}{\sqrt{8}+\sqrt{12}}$ is equal to

(a) $\sqrt{2}$ (b) 2 (c) 4 (d) 8

Sol : $\frac{\sqrt{32}+\sqrt{48}}{\sqrt{8}+\sqrt{12}} = \frac{4\sqrt{2}+4\sqrt{3}}{2\sqrt{2}+2\sqrt{3}} = \frac{4(\sqrt{2}+\sqrt{3})}{2(\sqrt{2}+\sqrt{3})} = \frac{4}{2} = 2$

Sol. b

16. Simplify $\sqrt{72}$ + $\sqrt{800}$ - $\sqrt{18}$

Sol: $\sqrt{72}$ + $\sqrt{800}$ - $\sqrt{18}$

= $\sqrt{6 \times 6 \times 2}$ + $\sqrt{2 \times 2 \times 2 \times 10 \times 10}$ - $\sqrt{3 \times 3 \times 2}$

= $6\sqrt{2} + 20\sqrt{2} - 3\sqrt{2} = (6 + 20 - 3)\sqrt{2} = 23\sqrt{2}$

17. State with reasons whether $\sqrt{20}$ x $\sqrt{45}$ is a surd or not?

Sol: We have $\sqrt{20}$ x $\sqrt{45} = \sqrt{20 \times 45}$ =$\sqrt{900}$

= $\sqrt{30 \times 30}$ = 30

Which is a rational number and therefore $\sqrt{20}$ x $\sqrt{45}$ is not a surd.

18. Simplify $(\sqrt{13} + \sqrt{5})(\sqrt{13} - \sqrt{5})$

Sol: $(\sqrt{13} + \sqrt{5})(\sqrt{13} - \sqrt{5}) = (\sqrt{13})^2 - (\sqrt{5})^2$

$[\therefore\ (a + b)(a - b) = a^2 - b^2]$

= 13 - 5 = 8

19. Simplify $\sqrt{125}$ X $\sqrt{5}$

Sol: $\sqrt{125}$ ×$\sqrt{5}$ = $(5^3)^{\frac{1}{2}}$ ×$(5)^{\frac{1}{2}}$ = $(5)^{\frac{3}{2}} \times (5)^{\frac{1}{2}}$

= $(5)^{\frac{3}{2}+\frac{1}{2}}$ $[a^m a^n = a^{m+n}]$

= $5^{\frac{4}{2}}$ = $5^2 = 25$

III. Multiple choice questions

1.Onsimplifying $8^3, 2^4$, we get

(a) 16^{17} (b) 2^{13} (c)2^{10} (d) 8^4

Sol: $8^3.2^4 = (2^3)^3 \,.\, 2^4 = 2^9 \,.\, 2^4 = 2^{9+4} = 2^{13}$

Sol. b

2. $(16)^{\frac{3}{4}}$ is equal to

(a) 2 (b) 4 (c) 8 (d) 16

Sol. c

3. $(125)^{\frac{-1}{3}}$ can be written as

(a) 5 (b) -5 (c) $\frac{1}{5}$ (d) none of these

Sol. c

4. $(36)^{\frac{3}{2}}$ is equal to

(a) 36 (b) 6 (c) 216 (d) 72

Sol. c

5. Simplified form of $3^{\frac{2}{3}}, 3^{\frac{1}{5}}$ is

(a) $3^{\frac{2}{15}}$ (b) $9^{\frac{2}{15}}$ (c) $3^{\frac{2}{3}}$ (d) $3^{\frac{13}{15}}$

Sol. d

6. Simplified value of $(16)^{\frac{-1}{4}} \times \sqrt[4]{16}$ *is*

(a) 16 (b) 4 (c) 1 (d) 0

$(16)^{-\frac{1}{4}} \times \sqrt[4]{16} = (16)^{-\frac{1}{4}} \times (16)^{-\frac{1}{4}}$

$= (16)^{\frac{-1}{4}+\frac{1}{4}} = (16)^0 = 1$

Sol. c

7. $\left(-\frac{1}{27}\right)^{\frac{-2}{3}}$ is equal to

(a) $8\left(\frac{1}{27}\right)^{\frac{-2}{3}}$ (b) 9 (c) $\frac{1}{9}$ (d) $27\sqrt{27}$

$\left(\frac{-1}{27}\right)^{\frac{-2}{3}} = \left(\frac{-1}{3^3}\right)^{\frac{-2}{3}} = (-1)^{\frac{-2}{3}} \times (3^{-3})^{\frac{-2}{3}}$

$= \{(-1)^3\}^{\frac{-2}{3}} \times 3^2 = 1 \times 9 = 9$

Sol. b

8. Which of the following is equal to x? [NCERT Exemplar]

(a) $x^{\frac{12}{7}} - x^{\frac{5}{7}}$ (b) $\sqrt[12]{(x^4)^{\frac{1}{3}}}$ (c) $(\sqrt{x^3})^{\frac{2}{3}}$ (d) $x^{\frac{12}{7}} \times x^{\frac{7}{12}}$

Sol. c

9. Find the value of $\frac{2^0+7^0}{5^0}$ [CBSE 2011]

Sol: We know that $a^0 = 1$

$\therefore \quad \frac{2^0+7^0}{5^0} = \frac{1+1}{1} = \frac{2}{1} = 2$

10. Find the value of $\sqrt{(3^{-2})}$ [CBSE 2011]

Sol: $\sqrt{(3^{-2})} = (3^{-2})^{\frac{1}{2}}$

$= 3^{-2x\frac{1}{2}}$ $[(a^m)^n = a^{mn}]$

$= 3^{-1}$

$= \frac{1}{3}$ $\left[a^{-m} = \frac{1}{a^m}\right]$

11. Which is the greatest among $\sqrt{2}, \sqrt[3]{4}$ *and* $\sqrt[4]{3}$? **[CBSE 2015]**

Sol: The order of the given surds are, 2,3 and 4 respectively.

$\therefore$ L.C.M. of 2,3 and 4 = 12

Now, $\sqrt{2} = \sqrt[12]{2^6} = \sqrt[12]{64}$

$\sqrt[3]{4} = \sqrt[12]{4^4} = \sqrt[12]{256}$

$\sqrt[4]{3} = \sqrt[12]{3^3} = \sqrt[12]{27}$

Clearly, $256 > 64 > 27$

$\Rightarrow \quad \sqrt[12]{256} > \sqrt[12]{64} > \sqrt[12]{27}$

$\Rightarrow \quad \sqrt[3]{4} > \sqrt{2} > \sqrt[4]{3}$

12. Find the value of $81^{0.16}\ x\ 81^{0.09}$ **[CBSE 2015]**

Sol: $81^{0.16} \times 81^{0.09} = 81^{0.16\,+\,0.09}$ $[a^m.a^n = a^{m+n}]$

$= (81)^{0.25} = (81)^{\frac{25}{100}} = (3^4)^{\frac{1}{4}}$

$= 3^{4\times\frac{1}{4}} = 3$

13. Find the value of $x^{a-b} \times x^{b-c} \times x^{c-a}$ **[CBSE 2016]**

Sol: $x^{a-b} \times x^{b-c} x x^{c-a} = x^{a-b+b-c+c-a}$ $[a^m.a^n.a^p = a^{m+n+p}]$

$= x^0 = 1$ $[a^0 = 1]$

14. Find the value of $\left[(16)^{\frac{1}{2}}\right]^{\frac{1}{2}}$ **[CBSE 2016]**

Sol: $\left[(16)^{\frac{1}{2}}\right]^{\frac{1}{2}} = \left[(4^2)^{\frac{1}{2}}\right]^{\frac{1}{2}} = \left(4^{2\times\frac{1}{2}}\right)^{\frac{1}{2}}$ $[(a^m)^n = a^{mn}]$

$= (4)^{\frac{1}{2}} = (2^2)^{\frac{1}{2}} = 2^{2\times\frac{1}{2}} = 2$

15. Find the value of $\left(\frac{64}{25}\right)^{\frac{-3}{2}}$ **[CBSE 2016]**

Sol: $\left(\frac{64}{25}\right)^{\frac{-3}{2}} = \left[\left(\frac{8^2}{5^2}\right)\right]^{-\frac{3}{2}} = \left[\left(\frac{8}{5^2}^{2}\right)\right]^{-\frac{3}{2}} = \left(\frac{8}{5}\right)^{-2\times\frac{3}{2}} = \left(\frac{8}{5}\right)^{-3}$

$= \left(\frac{5}{8}\right)^3 = \frac{125}{512}$

16. Simplify $\frac{7^{\frac{1}{3}}}{7^{\frac{1}{5}}}$.

Sol: $\frac{7^{\frac{1}{3}}}{7^{\frac{1}{5}}} = 7^{\left(\frac{1}{3}-\frac{1}{5}\right)} = 7^{\frac{5-3}{15}} = 7^{\frac{2}{15}}$

I Short Answer TypeQuestions

1. Express $3.\overline{2}$in the form $\frac{p}{q}$, where p and q are integers and q $\neq 0$.

Sol : Let $x = 3.\overline{2}$= 3.2222... ...(i)

Here, only one digit is repeating.

Multiplying both sides by 10, we get

$10\,x = 32.222 = 32.\overline{2}$ ...(ii)

Subtracting (i) from (ii), we get

$10x - x = 32.\overline{2} - 3.\overline{2} = 29$

$\Rightarrow\ 9x = 29$

$\therefore x = \frac{29}{9}$.

2. **Express $18.\overline{48}$ in the form of $\frac{p}{q}$, where p and q are integers and q $\neq 0$**

Sol : Let x= $18.\overline{48}$ = 18.484848… …(i)

Hero, two digits are repeating.

Multiplying both sides by 100, we get

100 x =1848.4848… = 1848. $\overline{48}$ …(ii)

Subtracting (i) from (ii), we get

100 x – x = 1848. $\overline{48}$ - $18.\overline{48}$

Or 99 x= 1830

Or $x = \frac{1830}{99} = \frac{610}{33}$

3. **Express $\frac{4}{7}$ in decimal form and state the kind of decimal expansion.**

Sol : $\frac{4}{7}$= 0.571428571428….= $\overline{0.571428}$

Therefore, the decimal expansion of the given rational number is non-terminating recurring (repeating).

4. **Find the rational number of the form $\frac{p}{q}$ corresponding to the decimal representation 0.222 ….., where p and q are integers and q $\neq 0$.**

Sol : Let x = 0.222 … = $0.\overline{2}$

Here, only one digit is repeating.

Multiplying both sides by 10, we get

$10x = 2.2222\ldots\ldots = 2.\overline{2} = 2 + 0.\overline{2} = 2 + x$

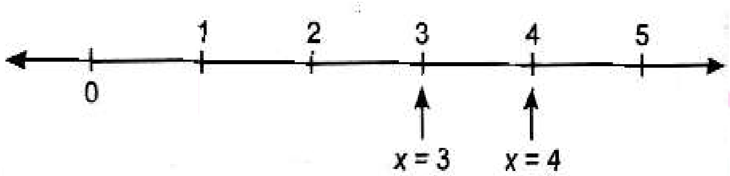
$\Rightarrow$ $10x - x = 2$

$\Rightarrow$ $9x = 2$

$\Rightarrow$ $x = \frac{2}{9}$

5. Represent the real numbers given by $2 < x < 5$ on the number line.

Sol : 3 and 4 are the real numbers which lie between 2and 5. Hence,



6. Represent$\sqrt{2}$ on the real number line

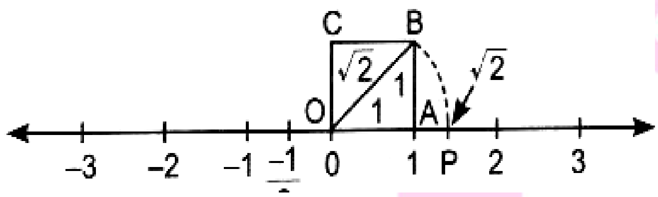
Sol : Using Pythagoras theorem,

$$\sqrt{2} = \sqrt{1^2 + 1^2}$$

$$\Rightarrow \quad OB = \sqrt{OA^2 + AB^2} = \sqrt{2}$$

Hence, take OA = 1 unit on the number line and AB = 1 unit, which is perpendicular to OA. With O as centre and OB as radius, we draw an arc to intersect the number line at P. Then P corresponds to $\sqrt{2}$ on the number lines as shown in figure .

Clearly, OP = OB = $\sqrt{2}$



7.Find an irrational number between $\frac{1}{7}$and $\frac{2}{7}$.Given that $\frac{1}{7} = 0.\overline{142857}$

Sol : Given $\quad \frac{1}{7} = 0.\overline{142857}$

$$\therefore \quad \frac{2}{7} = 2 \times 0.\overline{142857} = 0.\overline{285714}$$

One of the non-terminating non-recurring number

between$\frac{1}{7}$ and $\frac{2}{7}$ is 0.150150015000150000015.......

II Short Answer Type Questions

8. Examine whether $\sqrt{2}$ is rational or irrational number

Sol : Let us find the square root of 2 by division method.

We get $\sqrt{2} = 1.41421356$

	1.41421356…
1	2. $\overline{00}\ \overline{00}\ \overline{00}\ \overline{00}\ \overline{00}\ \overline{00}\ \overline{00}$ 1
24	100 96
281	400 281
2824	11900 11296
28282	60400 56564
282841	383600 282841
2828423	10075900 8485269
28284265	159063100 141421325
282842706	1764177500 1697056236
	67121264

Thus the process will neither terminate nor a block of digits will repeat in the process. Hence, $\sqrt{2}$ has a non – terminating decimal expansion

$\therefore \sqrt{2}$ is an irrational number

9. Represent $\sqrt{17}$ on number line

Sol : 17 can be written as

17 = 16 + 1 = $4^2 + 1^2$

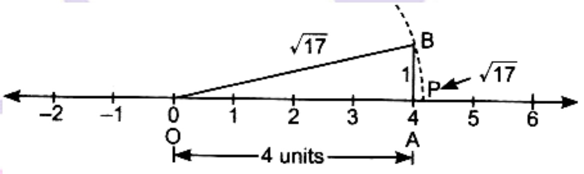
$\therefore \sqrt{17} = \sqrt{4^2 + 1^2} \implies OB = \sqrt{OA^2 + AB^2}$

$\therefore$ *On the number line, we mark*

$OA = 4$ *units*

AB = 1 unit and AB $\perp OA$ *at* A.

Using a compass with centre O and radius OB, draw an are intersecting the number line at the point P. Then point P corresponds to $\sqrt{17}$ on the number line as shown in figure.



10.Express 1.4191919…. in the form $\frac{p}{q}$, *where p and q are integers and q* $\neq 0$.

Sol : Let $x = 1.4191919 \ldots. = 1.4\overline{19}$

Multiplying both sides by 10. *we get*

$10x = 14.\overline{19}$

Here, two digits are repeated continuously, therefore, again multiplying both side by 100, we get

$$1000x = 1419.\overline{19} = 1405 + 14.\overline{19}$$

= 1405 + 10 x

$\implies \quad 1000x - 10x = 1405 \implies 990x = 1405$

$\implies \quad x = \frac{1405}{900} = \frac{281}{198}$

11. In the following equations, examine whether x , y and z represents rational or irrational number.

(i) $x^3 = 27$ (ii) $y^2 = 7$ (iii) $z^2 = 0.16$

Sol : (i) $x^3 = 27$

$$\Rightarrow \quad x = \sqrt[3]{27} = \sqrt[3]{3 \times 3 \times 3} = 3 = \frac{3}{1}$$

So, it is a rational number.

(ii) $y^2 = 7$

$$\Rightarrow \quad y = \sqrt{7} \neq \frac{p}{q}$$

So, it is an irrational number.

(iii) $z^2 = 0.16 = \frac{16}{100}$

$$\therefore \quad z = \sqrt{\frac{16}{100}} = \sqrt{\frac{4 \times 4}{10 \times 10}}$$

$$= \frac{4}{10} = \frac{2}{5} = \frac{p}{q}$$

Hence, it is a rational number,

12.State whether the following statements are true or false, Give reasons for your answers.

(i) Every whole number is a natural number.

(ii) Every integer is a rational number.

(iii) Every rational number is an integer.

Sol : (i) False, because whole numbers contains 0 but natural numbers does not, i.e. 0 is not a natural number.

(ii)True, because every integer can be expressed in the form $\frac{p}{q}$, $q = 1$

(iii) False, because $\frac{2}{5}$ is not an integer.

III. Short answer Type questions

1. Find the value of $\sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}}$, if $\sqrt{3} = 1.73$

Sol : Consider $\sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}}$, on rationalising the denominator by multiplying and dividing it by $\sqrt{2+\sqrt{3}}$ we get

$$\sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}} = \sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}}\; x\; \sqrt{\frac{2+\sqrt{3}}{2+\sqrt{3}}}$$

$$= \sqrt{\frac{(2+\sqrt{3})^2}{(2)^2-(\sqrt{3})^2}}$$

$$= \frac{2+\sqrt{3}}{1} = 2 + \sqrt{3} = 2 + 1.73 = 3.73$$

2. Simplify $\frac{6-4\sqrt{3}}{6+4\sqrt{3}}$ by rationalising the denominator

Sol : Here the denominator is $6 + 4\sqrt{3}$

Multiplying the numerator and denominator by its conjugate $(6 - 4\sqrt{3})$ we get

$$\frac{6-4\sqrt{3}}{6+4\sqrt{3}} = \left(\frac{6-4\sqrt{3}}{6+4\sqrt{3}}\right) \times \left(\frac{6-4\sqrt{3}}{6-4\sqrt{3}}\right) = \frac{(6-4\sqrt{3})^2}{(6)^2-(4\sqrt{3})^2}$$

$$= \frac{36-48\sqrt{3}+48}{36-48} \qquad [(a-b)^2 = a^2 - 2ab + b^2$$

$$= \frac{36-48\sqrt{3}}{-12} = \frac{12(7-4\sqrt{3})}{-12} = 4\sqrt{3} - 7$$

3. If $x = 3 + 2\sqrt{2}$, then find whether $x + \frac{1}{x}$ is rational or irrational

Sol : Given $x = 3 + 2\sqrt{2}$

$$\therefore \frac{1}{x} = \frac{1}{3+2\sqrt{2}} = \frac{1}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}}$$

$$= \frac{3-2\sqrt{2}}{(3)^2-(2\sqrt{2})^2} = \frac{3-2\sqrt{2}}{9-8} = 3 - 2\sqrt{2}$$

$$\therefore x + \frac{1}{x} = 3 + 2\sqrt{2} + 3 - 2\sqrt{2} = 6$$

Which is a rational number.

Hence, $x + \frac{1}{x}$ $for\ x = 3 + 2\sqrt{2}$ is a rational number.

4. Simplify $\sqrt[4]{81}$ - 8 $\left(\sqrt[3]{216}\right) + 15\left(\sqrt[5]{32}\right) + \sqrt{225}$.

Sol : Hence, $\sqrt[4]{81} = (81)^{\frac{1}{4}} = (3^4)^{\frac{1}{4}} = 3^{4 \times \frac{1}{4}} = 3$

$\sqrt[3]{216} = (216)^{\frac{1}{3}} = (6^3)^{\frac{1}{3}} = 6^{3 \times \frac{1}{3}} = 6$

$\sqrt[5]{32} = (32)^{\frac{1}{5}} = 2^{5 \times \frac{1}{5}} = 2$

$\sqrt{225} = (225)^{\frac{1}{2}} = (15^2)^{\frac{1}{2}} = 15^{2 \times \frac{1}{2}} = 15$

Hence, $\sqrt[4]{81} - 8\left(\sqrt[3]{216}\right) + 15\left(\sqrt[5]{32}\right) + \sqrt{225}$

$= 3 - 8 \times 6 + 15 \times 2 + 15$

$= 3 - 48 + 30 + 15 = 48 - 48 = 0$

5. Find the value of a and b, if $\frac{\sqrt{3}+1}{\sqrt{3}+1} = a + b\sqrt{3}$

Sol : Here the denominator is $\sqrt{3} + 1$

Multiplying the numerator and denominator by its conjugate $\sqrt{3} - 1$, *we get*

$\frac{\sqrt{3}-1}{\sqrt{3}+1} = \left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right) \times \left(\frac{\sqrt{3}-1}{\sqrt{3}-1}\right)$

$= \frac{(\sqrt{3}-1)^2}{(\sqrt{3})^2 - 1^2} = \frac{3+1-2\sqrt{3}}{3-1}$

$= \frac{4-2\sqrt{3}}{2} = \frac{2(2-\sqrt{3})}{2} = 2 - \sqrt{3}$

$\therefore\ 2 - \sqrt{3} = a + b\sqrt{3}$

Hence, on equating rational and irrational part both sides, we get a = 2, b = -1

6. If a = $\sqrt{2}$ + 1, find the value of $\left(a - \frac{1}{a}\right)^2$

Sol : Given $a = \sqrt{2} + 1$

$\therefore \quad \frac{1}{a} = \frac{1}{\sqrt{2}+1}$

$\Rightarrow \quad \frac{1}{a} = \left(\frac{1}{\sqrt{2}+1}\right) x \left(\frac{\sqrt{2}-1}{\sqrt{2}-1}\right)$ [Rationalising the denominator]

$= \frac{\sqrt{2}-1}{(\sqrt{2})^2 - 1} = \frac{\sqrt{2}-1}{2-1} = \sqrt{2} - 1$

$\therefore \quad a - \frac{1}{a} = (\sqrt{2}+1) - (\sqrt{2}-1)$

$= \sqrt{2} + 1 - \sqrt{2} + 1 = 2$

$\therefore \quad \left(a - \frac{1}{a}\right)^2 = 2^2 = 4$

IV. Short answer Type questions

7. Simplify $\frac{4+\sqrt{5}}{4-\sqrt{5}} + \frac{4-\sqrt{5}}{4+\sqrt{5}}$

Sol : $\frac{4+\sqrt{5}}{4-\sqrt{5}} + \frac{4-\sqrt{5}}{4+\sqrt{5}}$

$= \left(\frac{4+\sqrt{5}}{4-\sqrt{5}}\right) \times \left(\frac{4+\sqrt{5}}{4+\sqrt{5}}\right) + \left(\frac{4-\sqrt{5}}{4+\sqrt{5}}\right) \times \left(\frac{4-\sqrt{5}}{4-\sqrt{5}}\right)$

[Rationalising both denominator]

$= \frac{(4+\sqrt{5})^2}{(4)^2-(\sqrt{5})^2} + \frac{(4-\sqrt{5})^2}{(4)^2-(\sqrt{5})^2}$

$= \frac{16+5+8\sqrt{5}}{16-5} + \frac{16+5-8\sqrt{5}}{16-5}$

$= \frac{1}{11}[21+8\sqrt{5}+21-8\sqrt{5}] = \frac{42}{11}$

8. Simplify $3\sqrt{45} - \sqrt{125} + \sqrt{200} - \sqrt{50}$

Sol : Given $3\sqrt{45} - \sqrt{125} + \sqrt{200} - \sqrt{50}$

Now, $\sqrt{45} = \sqrt{5 \times 3 \times 3} = 3\sqrt{5}$

$\sqrt{125} = \sqrt{5 \times 5 \times 5} = 5\sqrt{5}$

$\sqrt{200} = \sqrt{2 \times 10 \times 10} = 10\sqrt{2}$

$\sqrt{50} = \sqrt{5 \times 5 \times 2} = 5\sqrt{2}$

$\therefore 3\sqrt{45} - \sqrt{125} + \sqrt{200} - \sqrt{50}$

$= 3 \times 3\sqrt{5} - 5\sqrt{5} + 10\sqrt{2} - 5\sqrt{2}$

$= 9\sqrt{5} - 5\sqrt{5} + 5\sqrt{2} = 4\sqrt{5} + 5\sqrt{2}$

9. If p = $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ and q=$\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$, then find p^2+q^2

Sol : Given p = $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

Multiplying and dividing R.H.S. by $\sqrt{3}$ - $\sqrt{2}$, we get

P = $\left(\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}\right) \times \left(\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}\right)$

= $\frac{(\sqrt{3}-\sqrt{2})^2}{(\sqrt{3})^2-(\sqrt{2})^2} = \frac{3\ +\ 2-2\sqrt{6}}{3-2}$ = 5 - 2$\sqrt{6}$

And q = $\left(\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}\right) \times \left(\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}\right)$

[Rationalizing the denominator]

On solving, we get

q = 5 + 2$\sqrt{6}$

Now pq = $(5-2\sqrt{6})(5+2\sqrt{6})$

= $(5)^2-(2\sqrt{6})^2 = 25-24 = 1$

and p + q = 5 - 2$\sqrt{6}$ + 5 + 2$\sqrt{6}$ = 10

Now $(p+q)^2 = 10^2$

$\Rightarrow$ $p^2+q^2+2pq = 100$

$\Rightarrow$ $p^2+q^2+2\times 1 = 100$

$\Rightarrow$ $p^2+q^2 = 100-2 = 98$

10. If x = 2 + $\sqrt{3}$, Find the value of $x^3 + \frac{1}{x^3}$

Sol : Given x = 2 + $\sqrt{3}$

$\therefore$ $\frac{1}{x} = \frac{1}{2+\sqrt{3}} = \left(\frac{1}{2+\sqrt{3}}\right) \times \left(\frac{2-\sqrt{3}}{2-\sqrt{3}}\right)$

[Rationalizing the denominator]

$\Rightarrow \frac{1}{x} = \frac{2-\sqrt{3}}{(2)^2-(\sqrt{3})^2} = \frac{2-\sqrt{3}}{4-3}$ =2- $\sqrt{3}$

$\therefore$ $x + \frac{1}{x}$ = 2 + $\sqrt{3}$ + 2 - $\sqrt{3}$ = 4

$\Rightarrow$ $\left(x+\frac{1}{x}\right)^3 = 4^3$

$\Rightarrow$ $x^3+\frac{1}{x^3}+3x\frac{1}{x}\left(x+\frac{1}{x}\right)$ = 64

$$\Rightarrow \quad x^3 + \frac{1}{x^3} + 3 \times 4 = 64$$

$$\Rightarrow \quad x^3 + \frac{1}{x^3} = 64 - 12 \Rightarrow x^3 + \frac{1}{x^3} + = 52$$

11. If $\sqrt{5}$ = 2.236 and $\sqrt{6}$ = 2.449, find the value of $\frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}} + \frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}$ [CBSE 2016]

Sol : Let $x = \frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}} = \left(\frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}}\right) \times \left(\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}\right)$

[Rationalizing the denominator]

$$\Rightarrow \quad x = \frac{(1+\sqrt{2})(\sqrt{5}-\sqrt{3})}{(\sqrt{5})^2 - (\sqrt{3})^2}$$

$$= \frac{\sqrt{5}-\sqrt{3}+\sqrt{10}-\sqrt{6}}{5-3}$$

$$= \frac{1}{2}(\sqrt{5}-\sqrt{3}+\sqrt{10}-\sqrt{6})$$

Again let $= \frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}$

Similarly, on rationalizing the denominator and solving, we get,

y = $\frac{1}{2}(\sqrt{5}+\sqrt{3}-\sqrt{10}-\sqrt{6})$

$\therefore \quad x + y$

= $\frac{1}{2}[\sqrt{5}-\sqrt{3}+\sqrt{10}-\sqrt{6}+\sqrt{5}+\sqrt{3}-\sqrt{10}-\sqrt{6}]$

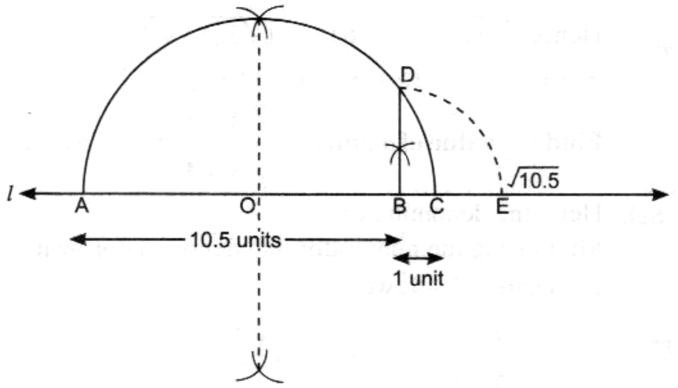
= $\frac{1}{2}[2\sqrt{5}-2\sqrt{6}] = \sqrt{5}-\sqrt{6}]$

= 2.236 - 2.449 = - 0.213

12. Represent $\sqrt{10.5}$ on the number line.

[CBSE 2016]

Sol.



Steps of Construction:

(i) Draw a line AB such that AB=10.5 units on the number line.

(ii) Extend the line *l* further from B up to C such that BC = 1 units

(iii) Find the mid-point of AC and mark it as O.

(iv) Draw a semicircle with centre O and radius OC.

(v) Draw a line perpendicular to AC passing through point B and cut the semicircle at D

(vi) Taking B as centre, draw an arc of radius BD which intersects the number line at E,

(vii) Point E represents $\sqrt{10.5}$ on the number line.

∴ BD = BE = $\sqrt{10.5}$ units, with B as zero.

13. Simplify $3\sqrt{45} - \frac{5}{2}\sqrt{\frac{1}{3}} + 4\sqrt{3}$

Sol : $3\sqrt{45} - \frac{5}{2}\sqrt{\frac{1}{3}} + 4\sqrt{3}$

$= 3\sqrt{3 \times 3 \times 5} - \frac{5}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} + 4\sqrt{3}$

$= 9\sqrt{5} - \frac{5}{6}\sqrt{3} + 4\sqrt{3}$

$= 9\sqrt{5} + \left(4 - \frac{5}{6}\right)\sqrt{3} = 9\sqrt{5} + \frac{19}{6}\sqrt{3}$

V. Short answer Type questions

1. If z = 0.064, then find the value of $\left(\frac{1}{z}\right)^{\frac{1}{3}}$ **[CBSE 2013]**

Sol : Given z= 0.064

$\therefore \quad \frac{1}{z} = \frac{1}{0.064} = \frac{1000}{64} = \left(\frac{10}{4}\right)^3$

So, $\left(\frac{1}{z}\right)^{\frac{1}{3}} = \left[\left(\frac{10}{4}\right)^3\right]^{\frac{1}{3}} = \left(\frac{10}{4}\right)^{3\times\frac{1}{3}}$ $\quad [(a^m)^n = a^{mn}]$

$= \frac{10}{4} = \frac{5}{2} = 2.5$

2. Simplify $\left[\frac{15^{\frac{1}{4}}}{9^{\frac{1}{4}}}\right]$ **[CBSE 2011]**

Sol : $\frac{15^{\frac{1}{4}}}{9^{\frac{1}{4}}} = \frac{(5\times3)^{\frac{1}{4}}}{(3^2)^{\frac{1}{4}}} = \frac{5^{\frac{1}{4}}\times3^{\frac{1}{4}}}{3^{2\times\frac{1}{4}}}$

$= 5^{\frac{1}{4}} \times 3^{\frac{1}{4}-\frac{1}{2}}$ $\quad \left[\frac{a^m}{a^n} = a^{m-n}\right]$

$= 5^{\frac{1}{4}} \times 3^{\frac{1-2}{4}} = = 5^{\frac{1}{4}} \times 3^{-\frac{1}{4}}$

$= \left(\frac{5}{3}\right)^{\frac{1}{4}}$ $\quad \left[a^{-m} = \frac{1}{a^m}\right]$

3. Evaluate $\left(\frac{32}{243}\right)^{-\frac{4}{5}}$ **[CBSE 2011]**

Sol : $\left(\frac{32}{243}\right)^{-\frac{4}{5}} = \left[\frac{2^5}{3^5}\right]^{-\frac{4}{5}} = \left[\left(\frac{2}{3}\right)^5\right]^{-\frac{4}{5}}$

$= \left(\frac{2}{3}\right)^{-5\times\frac{4}{5}}$ $\quad [(a^m)^n = a^{mn}]$

$= \left(\frac{2}{3}\right)^{-4} = \left(\frac{3}{2}\right)^4$ $\quad \left[a^{-m} = \frac{1}{a^m}\right]$

$= \frac{81}{16}$

4. Simplify $\sqrt[4]{\sqrt[3]{x^2}}$ and express the result in the exponent form of x.[CBSE 2011]

Sol : Given $\sqrt[4]{\sqrt[3]{x^2}}$

The given expression can be written as

$\sqrt[4]{\sqrt[3]{x^2}} = \left[(x^2)^{\frac{1}{3}}\right]^{\frac{1}{4}} = \left(x^{2\times\frac{1}{3}}\right)^{\frac{1}{4}}$ $[(a^m)^n = a^{mn}]$

$= x^{2\times\frac{1}{3}\times\frac{1}{4}} = x^{\frac{1}{6}}$

5. Find the value of $\frac{4}{(216)^{-\frac{2}{3}}} - \frac{1}{(256)^{-\frac{3}{4}}}$. [CBSE 2011]

Sol : Here, $(216)^{-\frac{2}{3}} = (6^3)^{-\frac{2}{3}} = 6^{-3\times\frac{2}{3}} = 6^{-2} = \frac{1}{6^2} = \frac{1}{36}$

And $\frac{1}{256^{\frac{-3}{4}}} = (256)^{\frac{3}{4}} = (4^4)^{\frac{3}{4}} = 4^{4\times\frac{3}{4}} = 4^3 = 64$

$\therefore \frac{4}{(216)^{-\frac{2}{3}}} - \frac{1}{(256)^{-\frac{3}{4}}} = \frac{4}{\frac{1}{36}} - 64$

$= 4 \times 36 - 64 = 144 - 64$

$= 80$

6. Simplify (i) $\left\{\left[(625)^{-\frac{1}{2}}\right]^{-\frac{1}{4}}\right\}^2$

(ii) $64^{-\frac{1}{3}}\left[64^{\frac{1}{3}} - 64^{\frac{2}{3}}\right]$

Sol : (i) $\left\{\left[(625)^{-\frac{1}{2}}\right]^{-\frac{1}{4}}\right\}^2 = (625)^{\left(-\frac{1}{2}\right)\times\left(-\frac{1}{4}\right)\times 2}$

$=(5^4)^{\frac{1}{4}} = 5^{4\times\frac{1}{4}} = 5$

(ii) $64^{-\frac{1}{3}}\left[64^{\frac{1}{3}} - 64^{\frac{2}{3}}\right] = (4^3)^{-\frac{1}{3}} \times \left[(4^3)^{\frac{1}{3}} - (4^3)^{\frac{2}{3}}\right]$

$= 4^{-3\times\frac{1}{3}} \times 4^{3\times\frac{1}{3}} - 4^{3\times\frac{2}{3}}$

$= 4^{-1}[4 - 4^2]$

$= \frac{1}{4}[4 - 16] = -\frac{12}{4} = -3$

VI. Short answer Type questions

7. Find the value of $\frac{4}{(216)^{-\frac{2}{3}}} + \frac{1}{(256)^{-\frac{3}{4}}} + \frac{2}{(243)^{-\frac{1}{5}}}$

Sol : $\frac{4}{(216)^{-\frac{2}{3}}} + \frac{1}{(256)^{-\frac{3}{4}}} + \frac{2}{(243)^{-\frac{1}{5}}}$

$= \frac{4}{(6^3)^{\frac{-2}{3}}} + \frac{1}{(2^8)^{-\frac{3}{4}}} + \frac{2}{(3^5)^{-\frac{1}{5}}}$

$= \frac{4}{6^{-3\times\frac{2}{3}}} + \frac{1}{2^{-8\times\frac{3}{4}}} + \frac{2}{3^{-5\times\frac{1}{5}}} = \frac{4}{6^{-2}} + \frac{1}{2^{-6}} + \frac{2}{3^{-1}}$

$= 4 \times 6^2 + 2^6 + 2 \times 3 = 4 \times 36 + 64 + 6$

$= 144 + 70 = 214$

8. Prove that $\frac{2^{30} + 2^{29} + 2^{28}}{2^{31} + 2^{30} - 2^{29}} = \frac{7}{10}$ [CBSE 2011]

Sol : L.H.S. $= \frac{2^{30} + 2^{29} + 2^{28}}{2^{31} + 2^{30} - 2^{29}} = \frac{2^{28+2} + 2^{28+1} + 2^{28}}{2^{29+2} + 2^{29+1} - 2^{29}}$

$= \frac{2^{28}.2^2 + 2^{28}.2 + 2^{28}.1}{2^{29}.2^2 + 2^{29}.2 - 2^{29}.1} = \frac{2^{28}(2^2+2+1)}{2^{29}(2^2+2-1)}$

$= \frac{4+2+1}{2^{29-28}(4+2+1)} \left[a^m = \frac{1}{a^{-m}} \text{ and } a^m.a^{-n} = a^{m-n}\right]$

$= \frac{7}{2(5)} = \frac{7}{10}$ = R.H.S Hence Proved.

9. Simplify $\left[5[8^{\frac{1}{3}} + 27^{\frac{1}{3}}]\right]^{\frac{1}{4}}$ [CBSE 2011]

Sol : $\left[5\left[8^{\frac{1}{3}} + 27^{\frac{1}{3}}\right]\right]^{\frac{1}{4}} = \left[\left[5(2^3)^{\frac{1}{3}} + (3^3)^{\frac{1}{3}}\right]\right]^{\frac{1}{4}}$

$= \left[5\left(2^{3\times\frac{1}{3}} + 3^{3\times\frac{1}{3}}\right)\right]^{\frac{1}{4}}$

$= [5(2+3)]^{\frac{1}{4}} = (5^2)^{\frac{1}{4}}$

$= 5^{2\times\frac{1}{4}} = 5^{\frac{1}{2}} = \sqrt{5}$

10. Simplify $8^{\frac{2}{3}} - \sqrt{9} \times 10^0 + \left(\frac{1}{144}\right)^{\frac{-1}{2}}$. **[CBSE 2011]**

Sol : $8^{\frac{2}{3}} - \sqrt{9} \times 10^0 + \left(\frac{1}{144}\right)^{\frac{1}{2}}$

$= (2^3)^{\frac{2}{3}} - \sqrt{3^2} \times 1 + \left(\frac{1}{12^2}\right)^{-\frac{1}{2}}$ $[a^0 = 1]$

$= 2^{3\times\frac{2}{3}} - (3^2)^{\frac{1}{2}} + \frac{1}{12^{-2\times\frac{1}{2}}}$

$= 2^2 - 3 + \frac{1}{12^{-1}} = 4 - 3 + 12 = 16 - 3 = 13$

11. Simplify $\sqrt[4]{81\, x^8 y^4\, z^{16}}$ **[CBSE 2014&15]**

Sol : $\sqrt[4]{81\, x^8\, y^4\, z^{16}} = (81\, x^8 y^4 z^{16})^{\frac{1}{4}}$

$= (81)^{\frac{1}{4}} \times (x^8) \times (y^4)^{\frac{1}{4}} \times (z^{16})^{\frac{1}{4}}$

$= 3^{4\times\frac{1}{4}} \times x^{8\times\frac{1}{4}} \times y^{4\times\frac{1}{4}} \times z^{16\times\frac{1}{4}}$ $[(a^m)^n = a^{mn}]$

$= 3 \times x^2 \times y \times z^4 = 3x^2\, y\, z^4$

12. Simplify $\frac{9^{\frac{1}{3}} \times 27^{-\frac{1}{2}}}{3^{\frac{1}{6}} \times 3^{-\frac{2}{3}}}$ **[CBSE 2016]**

Sol : $\frac{9^{\frac{1}{3}}\ X\ 27^{-\frac{1}{2}}}{3^{\frac{1}{6}}\ X\ 3^{-\frac{2}{3}}} = \frac{(3^2)^{\frac{1}{3}}\ X\ (3^2)^{-\frac{1}{2}}}{3^{\frac{1}{6}-\frac{2}{3}}}$

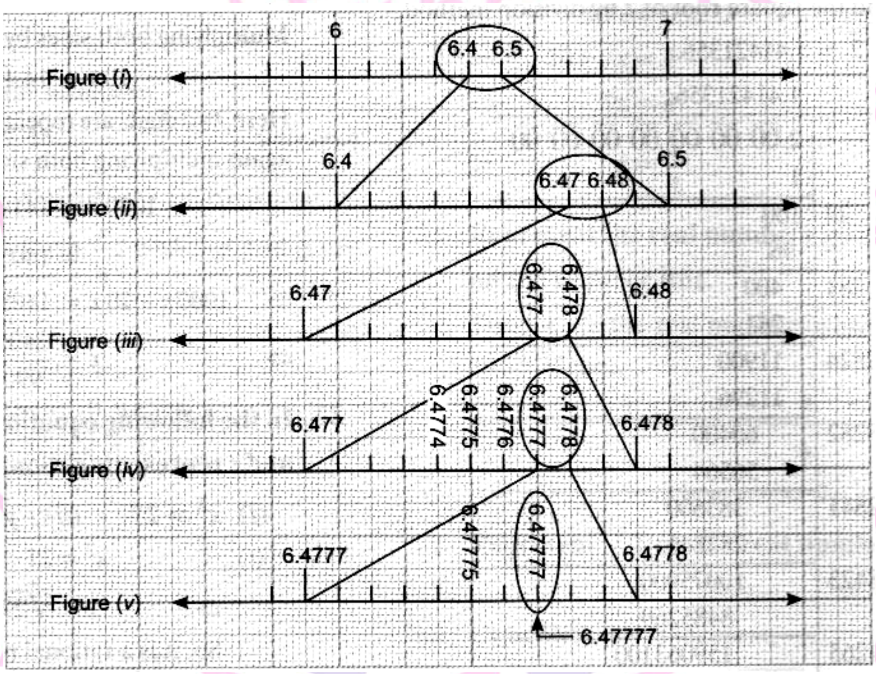
$= \frac{3^{\frac{2}{3}}\ X\ 3^{-\frac{3}{2}}}{3^{-\frac{3}{6}}} = \frac{3^{\frac{2}{3}-\frac{3}{2}}}{3^{-\frac{1}{2}}} = \frac{3^{-\frac{5}{6}}}{3^{-\frac{1}{2}}}$

$= 3^{-\frac{5}{6}}.\, 3^{\frac{1}{2}} = 3^{\frac{1}{2}-\frac{5}{6}} = 3^{\frac{3-5}{6}} = 3^{-\frac{2}{6}} = 3^{-\frac{1}{3}} = \frac{1}{\sqrt[3]{3}}$

I. Long answer choice questions

1. Visualise the representation of $6.4\overline{7}$ on the number line up to 5 decimal places, that is up to 6.47777. Draw figure only.

Sol :



2. Express $1.3\overline{2} + 0.\overline{35}$ as a fraction in simplest form.

Sol : Let $x = 1.3\overline{2}$ and $y = 0.\overline{35}$

(i) $Consider\ x = 1.3\overline{2} = 1.32222 \ldots$

$\Longrightarrow \quad 10x = 13.222 \ldots. = 13.\overline{2}$ ....(i)

$\Longrightarrow \quad 100x = 132.\overline{2}$ ....(ii)

$Subtracting\ (i) from\ (ii), we\ get$

$100x - 10x = 132.\overline{2} - 13.\overline{2}$

$90x = 119$

$x = \frac{119}{90}$

(ii) $Consider\ y = 0.\overline{35} = 0.353535 \ldots..$ .... (iii)

$\Longrightarrow \quad 100y = 35.3535 \ldots 35.\overline{35}$ ......

$\therefore \quad 100y - y = 35.\overline{35}$ - $0.\overline{35}$....... (iv)

$Subtracting\ (iii) from\ (iv), \quad we\ get$

$99\,y = 35$

$y = \frac{35}{99}$

Therefore,

$$1.3\overline{2} + 0.\overline{35} = x + y = \frac{119}{90} + \frac{35}{99} = \frac{1309 + 350}{90 \times 11}$$

$$= \frac{1659}{90 \times 11} = \frac{553}{330}$$

II. Long answer choice questions

1. **Evaluate $\frac{15}{\sqrt{10}+\sqrt{20}+\sqrt{40}-\sqrt{5}-\sqrt{80}}$ when it is given that $\sqrt{5}$ = 2.2 and $\sqrt{10}$ = 3.2**

[CBSE 2013]

Sol : Consider the denominator

$\sqrt{10} + \sqrt{20} + \sqrt{40} - \sqrt{5} - \sqrt{80}$

= $\sqrt{10} + \sqrt{5 \times 2 \times 2} + \sqrt{2 \times 2 \times 2 \times 5} - \sqrt{5} - \sqrt{4 \times 4 \times 5}$

= $\sqrt{10} + 2\sqrt{5} + 2\sqrt{10} - \sqrt{5} - 4\sqrt{5}$

= $3\sqrt{10} + 2\sqrt{5} - 5\sqrt{5} = 3\sqrt{10} - 3\sqrt{5}$

= $3(\sqrt{10} - \sqrt{5})$

$\therefore \frac{15}{\sqrt{10}+\sqrt{20}+\sqrt{40}-\sqrt{5}-\sqrt{80}}$

= $\frac{15}{3(\sqrt{10}-\sqrt{5})}$ = $\frac{5}{\sqrt{10}-\sqrt{5}}$

Multiplying and dividing by the conjugate of $\sqrt{10} - \sqrt{5}, i.e., \sqrt{10} + \sqrt{5}$, we get

$\frac{15}{\sqrt{10}+\sqrt{20}+\sqrt{40}-\sqrt{5}-\sqrt{80}}$

= $\left(\frac{15}{\sqrt{10}-\sqrt{5}}\right) \times \left(\frac{\sqrt{10}+\sqrt{5}}{\sqrt{10}+\sqrt{5}}\right)$

= $\frac{5(\sqrt{10}+\sqrt{5})}{(\sqrt{10})^2-(\sqrt{5})^2}$ [(a + b) (a - b) = $a^2 - b^2$]

= $\frac{5(\sqrt{10}+\sqrt{5})}{10-5}$ = $\sqrt{10}$ + $\sqrt{5}$ = 3.2 + 2.2 = 5.4

2. If $a = \frac{\sqrt{10}+\sqrt{5}}{\sqrt{10}-\sqrt{5}}$ **and** $b = \frac{\sqrt{10}-\sqrt{5}}{\sqrt{10}+\sqrt{5}}$, **then show**

That $\sqrt{a} = \sqrt{b} - 2\sqrt{ab} = 0.$ [CBSE 2014]

Sol : $\sqrt{a} = \sqrt{\frac{\sqrt{10}+\sqrt{5}}{\sqrt{10}-\sqrt{5}}}$

$= \sqrt{\left(\frac{\sqrt{10}+\sqrt{5}}{\sqrt{10}-\sqrt{5}}\right)} \times \sqrt{\left(\frac{\sqrt{10}+\sqrt{5}}{\sqrt{10}+\sqrt{5}}\right)}$

[Rationalizing the denominator]

$= \sqrt{\frac{(\sqrt{10}+\sqrt{5})^2}{(\sqrt{10})^2-(\sqrt{5})^2}}$

$[\because (a-b)(a+b) = a^2 - b^2]$

$= \frac{\sqrt{10}+\sqrt{5}}{\sqrt{10-5}}$

$\therefore \sqrt{a} = \frac{\sqrt{10}+\sqrt{5}}{\sqrt{5}}$

Similarly, $\sqrt{b} = \sqrt{\frac{\sqrt{10}-\sqrt{5}}{\sqrt{10}+\sqrt{5}}}$

After rationalizing the denominator, we get

$\sqrt{b} = \frac{\sqrt{10}-\sqrt{5}}{\sqrt{5}}$

And $\sqrt{a,b} = \sqrt{a} \times \sqrt{b}$

$= \left(\frac{\sqrt{10}+\sqrt{5}}{\sqrt{5}}\right) \times \left(\frac{\sqrt{10}-\sqrt{5}}{\sqrt{5}}\right)$

$= \frac{(\sqrt{10})^2-(\sqrt{5})^2}{(\sqrt{5})^2} = \frac{10-5}{5} = \frac{5}{5} = 1$

$\therefore$ L.H.S. = $\sqrt{a} - \sqrt{b} - 2\sqrt{ab}$

$= \left(\frac{\sqrt{10}+\sqrt{5}}{\sqrt{5}}\right) \times \left(\frac{\sqrt{10}-\sqrt{5}}{\sqrt{5}}\right) - 2 \times 1$

$= \frac{1}{\sqrt{5}}(\sqrt{10}+\sqrt{5}-\sqrt{10}+\sqrt{5})$-2

$= \frac{2\sqrt{5}}{\sqrt{5}} - 2 = 2 - 2$

= 0 = R.H.S. Hence proved.

3. If $x = \frac{\sqrt{2}+1}{\sqrt{2}-1}$ any y = $\frac{\sqrt{2}-1}{\sqrt{2}+1}$, find the value of $x^2 + y^2 + xy$

[CBSE 2014]

Sol : Consider $x = \frac{\sqrt{2}+1}{\sqrt{2}-1}$

$= \left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right) x \left(\frac{\sqrt{2}+1}{\sqrt{2}+1}\right)$

[Rationalising the denominator]

$= \frac{(\sqrt{2}+1)^2}{(\sqrt{2})^2 - 1^2} = \frac{2+1+2\sqrt{2}}{2-1}$

$= 3 + 2\sqrt{2}$

Similarly, y = $3 - 2\sqrt{2}$

Now $xy = (3 + 2\sqrt{2})(3 - 2\sqrt{2})$

$= (3)^2 - (2\sqrt{2})^2 = 9 - 8 = 1$

And $x + y = (3 + 2\sqrt{2}) + (3 - 2\sqrt{2}) = 6$

Squaring both sides, we get

$(x+y)^2 = 36 \implies x^2 + y^2 + 2xy = 36$

$\implies x^2 + y^2 + 2 \times 1 = 36 \implies x^2 + y^2 = 34$

Hence, $x^2 + y^2 + xy = 34 + 1 = 35$

4. Simplify $\sqrt{\frac{\sqrt{20}+\sqrt{11}}{\sqrt{20}-\sqrt{11}}}$. **[CBSE2014]**

Rationalisingthe denominator, we get

Sol : $\sqrt{\frac{\sqrt{20}+\sqrt{11}}{\sqrt{20}-\sqrt{11}}} = \sqrt{\frac{\sqrt{20}+\sqrt{11}}{\sqrt{20}-\sqrt{11}}} \times \sqrt{\frac{\sqrt{20}+\sqrt{11}}{\sqrt{20}+\sqrt{11}}}$

$= \sqrt{\frac{(\sqrt{20}+\sqrt{11})^2}{(\sqrt{20})^2 - (\sqrt{11})^2}} = \frac{\sqrt{20}+\sqrt{11}}{\sqrt{20-11}} = \frac{\sqrt{20}+\sqrt{11}}{\sqrt{9}}$

$= \frac{1}{3}(\sqrt{20}+\sqrt{11})$

5. If $x + \frac{1}{x} = \sqrt{3}$, find the value of $x^3 + \frac{1}{x^3}$ **[CBSE 2016]**

Sol : Given $x + \frac{1}{x} = \sqrt{3}$

$\Rightarrow \quad \left(x + \frac{1}{x}\right)^3 = \left(\sqrt{3}\right)^3$

[Cubing both sides]

$\Rightarrow \; x^3 + \frac{1}{x^3} + 3x.\frac{1}{x}\left(x + \frac{1}{x}\right) = 3^{\frac{3}{2}}$

$\Rightarrow \; x^3 + \frac{1}{x^3} + 3\sqrt{3} = 3\sqrt{3^3}$

$= \sqrt{3 \times 3 \times 3}$

$\Rightarrow \; x^3 + \frac{1}{x^3} + 3\sqrt{3} = 3\sqrt{3}$

$\Rightarrow \; x^3 + \frac{1}{x^3} = 3\sqrt{3} - 3\sqrt{3}$

$\Rightarrow \; x^3 + \frac{1}{x^3} = 0$

6. If a = 7 – 4$\sqrt{3}$, find the value of $\sqrt{a}$ + $\frac{1}{\sqrt{a}}$. **[CBSE 2011, 2014, 2016:HOTS]**

Sol : Given a = 7 – 4$\sqrt{3}$,

$\therefore \; \frac{1}{\sqrt{a}} = \frac{1}{7 - 4\sqrt{3}}$

= $\left(\frac{1}{7 - 4\sqrt{3}}\right) \times \left(\frac{7 + 4\sqrt{3}}{7 + 4\sqrt{3}}\right)$

[Rationalising the denominator]

$\Rightarrow \; \frac{1}{a} = \frac{7 + 4\sqrt{3}}{7^2 - \left(4\sqrt{3}\right)^2}$

$[\because (a-b)(a+b) = a^2 - b^2]$

= $\frac{7 + 4\sqrt{3}}{49 - 48}$ = 7 + 4$\sqrt{3}$

$\therefore a + \frac{1}{a} = 7 - 4\sqrt{3} + 7 + 4\sqrt{3} = 14$

Now $\left(\sqrt{a} + \frac{1}{\sqrt{a}}\right)^2$ = a + $\frac{1}{a} + 2.a.\frac{1}{a}$

= 14 + 2 =16

$\therefore \sqrt{a} + \frac{1}{\sqrt{a}} = \sqrt{16} = 4$

7. **Prove that** $\frac{1}{3-\sqrt{8}}-\frac{1}{\sqrt{8}-\sqrt{7}}+\frac{1}{\sqrt{7}-\sqrt{6}}-\frac{1}{\sqrt{6}-\sqrt{5}}+\frac{1}{\sqrt{5}-2}=5.$

Sol : $\frac{1}{3-\sqrt{8}}-\frac{1}{\sqrt{8}-\sqrt{7}}+\frac{1}{\sqrt{7}-\sqrt{6}}-\frac{1}{\sqrt{6}-\sqrt{5}}+\frac{1}{\sqrt{5}-2}=5.$

$= \left[\frac{1}{3-\sqrt{8}} \times \frac{3+\sqrt{8}}{3+\sqrt{8}}\right] - \left[\frac{1}{\sqrt{8}-\sqrt{7}} \times \frac{\sqrt{8}+\sqrt{7}}{\sqrt{8}+\sqrt{7}}\right] + \left[\frac{1}{\sqrt{7}-\sqrt{6}} \times \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}}\right] - \left[\frac{1}{\sqrt{6}-\sqrt{5}} \times \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}+\sqrt{5}}\right] + \left[\frac{1}{\sqrt{5}-2} \times \frac{\sqrt{5}+2}{\sqrt{5}+2}\right]$

$= \left[\frac{3+\sqrt{8}}{9-8}\right] - \left[\frac{\sqrt{8}+\sqrt{7}}{8-7}\right] + \left[\frac{\sqrt{7}+\sqrt{6}}{7-6}\right] - \left[\frac{\sqrt{6}+\sqrt{5}}{6-5}\right] + \left[\frac{\sqrt{5}+2}{5-4}\right]$

$= 3+\sqrt{8}-\sqrt{8}-\sqrt{7}+\sqrt{7}+\sqrt{6}-\sqrt{6}-\sqrt{5}+\sqrt{5}+2 = 5$

8. **Rationalise the denominator** $\frac{4}{2+\sqrt{3}+\sqrt{7}}$ [CBSE 2011]

Sol : $\frac{4}{2+\sqrt{3}+\sqrt{7}} = \frac{4}{(2+\sqrt{3})+\sqrt{7}} \times \frac{(2+\sqrt{3})-\sqrt{7}}{(2+\sqrt{3})-\sqrt{7}} = \frac{4(2+\sqrt{3})-\sqrt{7}}{(2+\sqrt{3})^2-(\sqrt{7})^2}$ [Using (a + b) (a - b) = a^2-b^2]

$= \frac{4(2+\sqrt{3}-\sqrt{7})}{4+3+4\sqrt{3}-7}$

$= \frac{4(2+\sqrt{3}-\sqrt{7})}{7+4\sqrt{3}-7} = \frac{4(2+\sqrt{3}-\sqrt{7})}{4\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{\sqrt{3}(2+\sqrt{3}-\sqrt{7})}{3} = \frac{2\sqrt{3}+3-\sqrt{21}}{3} = \frac{1}{3}[3+2\sqrt{3}-\sqrt{21}]$

III. Long answer choice questions

1. **If x is a positive real number and the exponents are rational numbers, then simplify.**

$\left(\frac{x^b}{x^c}\right)^{b+c-a} \times \left(\frac{x^c}{x^a}\right)^{c+a-b} \times \left(\frac{x^a}{x^b}\right)^{a+b-c}$ [CBSE 2011&2016]

Sol : Given $\left(\frac{x^b}{x^c}\right)^{b+c-a} \times \left(\frac{x^c}{x^a}\right)^{c+a-b} \times \left(\frac{x^a}{x^b}\right)^{a+b-c}$

$= \frac{(x^b)^{(b+c-a)}}{(x^b)^{a+b-c}} \times \frac{(x^c)^{c+a-b}}{(x^c)^{b+c-a}} \times \frac{(x^a)^{a+b-c}}{(x^a)^{c+a-b}}$

$= \frac{(x)^{b^2+bc-ab}}{(x)^{ab+b^2-bc}} \times \frac{(x)^{c^2+ca-bc}}{(x)^{bc+c^2-ac}} \times \frac{(x)^{a^2+ab-ac}}{(x)^{ac+a^2-ab}}$

$= \frac{(x)^{b^2+bc-ab+c^2+ac-bc+a^2+ab-ac}}{(x)^{ab+b^2-bc+bc+c^2-ac+ac+a^2-ab}}$ [$\because x^m \times x^n \times x^p = x^{m+n+p}$]

$= \frac{(x)^{a^2+b^2+c^2}}{(x)^{a^2+b^2+c^2}}$

$= (x)^{(a^2+b^2+c^2)-(a^2+b^2+c^2)}$ $\left[\frac{x^m}{x^n} x^{m-n}\right]$

$= x^0 = 1$ $[x^0 = 1]$

2. Simplify $\left(\frac{81}{16}\right)^{-\frac{3}{4}} \times \left[\left(\frac{25}{4}\right)^{-\frac{3}{2}} \div \left(\frac{5}{2}\right)^{-3}\right]$

Sol : $\left(\frac{81}{16}\right)^{-\frac{3}{4}} \times \left[\left(\frac{25}{4}\right)^{-\frac{3}{2}} \div \left(\frac{5}{2}\right)^{-3}\right]$

= $\left[\left(\frac{3}{2}\right)^{4}\right]^{-\frac{3}{4}} \times \left\{\left[\left(\frac{5}{2}\right)^{2}\right]^{-\frac{3}{2}} \div \left(\frac{5}{2}\right)^{-3}\right\}$

= $\left(\frac{3}{2}\right)^{-4\times\frac{3}{4}} \times \left[\left(\frac{5}{2}\right)^{-2\times\frac{3}{2}} \div \left(\frac{5}{2}\right)^{-3}\right]$

= $\left(\frac{3}{2}\right)^{-3} \times \left[\left(\frac{5}{2}\right)^{-3} \div \left(\frac{5}{2}\right)^{-3}\right]$

= $\left(\frac{2}{3}\right)^{3} \times \left[\left(\frac{5}{2}\right)^{-3+3}\right]$ $\quad\because \left[\frac{a^m}{a^n} a^{m-n}\right]$

= $\frac{2^3}{3^3} \times \left(\frac{5}{2}\right)^{0}$

= $\frac{8}{27} \times 1$

= $\frac{8}{27}$ $\quad [a^0 = 1]$